5 – 13 mars 2020.

A l'automne 2019 nous avons envoyé aux revues « cotées » un article (Annexe B) présentant une interprétation alternative de la solution mathématique de l'équation introduite par Einstein en 1915 par le mathématicien Karl Schwarzschild en 1916. Un article qui s'inscrit dans la ligne des vidéos Janus 22-1 à Janus 22-8.

On envoie d'abord aux deux revues les plus cotées : Physical Review D et Physical Letters. Ces deux revues formulent immédiatement un rejet dénué de tout argument scientifique, ne comportant qu'un seul qualificatif : « Non suitable » « ne convient pas », ou « pas convenable »). Inutile d'insister.

Nous démarchons alors après de la revue Modern Physics Letters A dans laquelle j'ai pu publier plusieurs articles (à grand peine) depuis 1988.

La première version de cet article constitue l'annexe A de ce document. Il faut que je retrouve les critiques formulées (en anglais) par un referee anonyme. Celui-ci demande au passage à ce que cinq ou six papiers composés par des chercheurs Indiens soient mentionnés. Ce qui laisse à penser que ce referee pourrait être un indien. Possible.

Que dit ce referee ? Il demande comment nous pouvons invoquer l'existence de ce qu'il croit être une « nouvelle solution » de cette équation, dans les conditions de symétrie considérées (symétrie sphérique, stationnarité) alors que Birkhoff a montré, dans les années vingt, que cette solution était unique.

Il y a là une incompréhension. Cela procède-t-il d'une lecture trop sommaire de l'article ou de son manque de clarté ? Peu importe. Je reprends cette rédaction en essayant d'être le plus clair possible et en écrivant d'emblée qu'il ne s'agit pas d'une nouvelle solution mais d'une interprétation de la solution de Schwarzschild de 1916 à travers un changement de variable.

Cette seconde rédaction est donnée dans l'annexe C.

Très vite le referee (ou un autre referee ?) demande à ce que les points forts du papier soient bien dégagés et que l'article soit plus concis. Demande parfaitement raisonnable.

Je reprends la rédaction selon la ligne demandée et nous renvoyons ce papier à la revue (Annexe D).

Quelques heures plus tard (lundi 2 mars à 11 heures) une décision de rejet est formulée, exempte de toute argumentation scientifique. La voici :

> Nous avons le regret de vous informer que l'article que vous avez soumis à notre revue intitulé « Réinterprétation de la solution stationnaire et à symétrie sphérique de l'équation d'Einstein ». ne correspond pas à notre politique éditoriale et à notre ligne de conduite académique. Nous rejetons donc votre artucle en vous souhaitant un meilleur résultat et soumettant votre article à des journaux plus adéquats.

Nous n'avons pas d'explication quant à la volte face du correspondant de la revue (mais est-ce le même ?). On peut se perdre en conjecture.

Nous avons adressé le 4 mars 2020 à la revue le message ci-après et nous vous tiendrons au courant de sa réaction.

> - I object. It is clear that the person who made this summary and absurd decision simply did not read this third version of our article and cannot be the referee who, after reading the second version of our article, had asked for its strong points to be made clear, in a more concise manner, which we did. We ask for a serious examination of this third version by a competent referee. If this decision of rejection were to be upheld, then your newspaper would never have published Einstein and Rosen's 1935 article, built, like ours, on a simple change of variable applied to the solution published by Karl Schwarzschild in 1916.
>
> - Nous protestons. Il est clair que la personne qui a pris cette décision sommaire et absurde n'a simplement pas lu cette troisième version de notre article et ne peut pas être le referee qui, après avoir lu la deuxième version de notre article, avait demandé que ses points forts soient mis en évidence, de manière plus concise, ce que nous avons fait. Nous demandons un examen sérieux de cette troisième version par un referee compétent. Si cette décision de rejet devait être maintenue, votre journal n'aurait jamais publié l'article d'Einstein et Rosen de 1935, construit, comme le nôtre, sur un simple changement de variable appliqué à la solution publiée par Karl Schwarzschild en 1916.

Au passage je tiens à préciser que toutes les demandes que j'ai pu adresser depuis 2013, depuis 7 ans à tous les laboratoires concernés, traduisent des propositions de séminaires, sont restées sans réponse. La réédition de ces démarches début 2020 s'est traduite par la même absence de réponse. Même chose pour des courriers adressés aux académiciens Thibaud Damour et Etienne Ghys (Secrétaire perpétuel) à l'automne 2020.

Explication d'un tel mutisme ? Il en est une possible, que l'on peut lire dans les mails que m'a adressé un certain Jean Staune, fondateur de l'UIP, Université Interdisciplinaire de Paris. Courriers suivis d'un échange téléphonique de 90 minutes (c'est lui m'a appelé). Voir l'annexe A

Un internaute voit dans le fait que le document envoyé ait été un pdf et non un document en latex la raison de son rejet. Nous ne pensons pas que cela soit l'argument. Mais cette transcription en larex ne nous pose pas de problème. Nous allons doubler notre dernier message par l'envoi du document dans ce format.

ANNEXE A : Extraits des mails de Jean Staune

Janvier 2020

Sa thèse : Mes travaux ne sont que l'expression du délire logique du malade mental que je suis.

Jean Staune, fondateur de l'UIP
Université Interdisciplinaire Paris
s'intitule «Philosophe chrétien»

Pour tout scientifique « normalement constitué » répondre à tes lettres, même une simple réponse de politesse serait une insulte à la raison et à la science.

Voilà pourquoi tu ne reçois jamais de réponse a toutes ses lettres dont tu envoies des copies à tes amis aux quatre coins du monde ou plutot aux quatre coins des académies…

Ta folie, ta fantaisie sont d'autres faces de ton génie.

Bien sur que tu es un Génie !

Et Bien sur que tu es FOU !

Je te l'ai déjà dit il a plus de 10 ans dans un café de la gare du Nord !

Je ne suis pas médecin mais le diagnostic de ta maladie est si facile à faire !!

Tu souffres de la même maladie que John Nash (que j'ai eu la chance de rencontrer) : le délire logique

Oui tu as 82 ans tu es assez malade, tu n'as plus que quelques années pour réaliser ce que tu te caches à toi même depuis tant de décennies !Et l'âge rend la chose encore plus difficile.

Je ne crois pas du tout que le modèle Janus soit capable de faire une seule prédiction qui serait vérifiée par les observations, là où la science actuelle ne serait pas capable de l'expliquer.

J'avais lu à l'époque ton papier « Dark Matter contre Twin Matter » et cela ne m'avait pas semblé très convaincant, en tout cas pas assez pour abandonner l'explication simple par la Dark Matter. **Je serai prêt à croire à la validité d'un ou plusieurs de ces points le jour où un seul scientifique, un seul, aura publié un seul article, un seul, dans une revue à référés de bon niveau disant : « le modèle Janus explique effectivement une anomalie observationnelle que j'ai faite et pour laquelle je n'avais pas d'explication ». C'est comme ça que fonctionne la science (tu es sensé le savoir…).**

Si cet événement se produit un jour (ce que j'espère pour toi), je serai prêt à croire non seulement à un mais à plusieurs de ces points. Pour l'instant on est dans « l'auto célébration » et « l'auto vérification » et je refuse absolument (tu m'en excuseras) de te croire sur parole sur un seul de ces « avantages » même si tu as les meilleures équations du monde pour accompagner tes affirmations…

Thibault Damour a fait l'effort de t'envoyer une lettre recommandée pour te signaler une faille dans le modèle Janus. Si c'est bien le cas il faut lui décerner immédiatement **le prix Nobel de l'ouverture d'esprit** !!!!!

C'est simplement incroyable que quelqu'un du niveau de Damour prennent la peine d'analyser tes travaux en sachant très bien qui tu es.

Toujours si j'ai bien compris, tu as admis la réalité de la faille qu'il a mentionnée, mais tu as dit qu'elle était mineure et que tu l'avais immédiatement corrigée et que tu avais renvoyé un nouveau modèle. Et que depuis Damour fait la sourde oreille, refuse absolument de commenter ce nouveau modèle corrigé, et exact selon toi. Et c'est pour cela que tu écris des lettres scandalisées à toute la planète (ou presque).

As-tu seulement pensé, juste à titre d'hypothèse, que l'attitude de Damour pouvait être absolument juste et rationnelle et donc en rien scandaleuse ??

Le petit problème c'est qu'il est possible que cette correction soit aussi fausse que ton modèle précédent et que Damour n'ait pas envie de perdre son temps pour te signaler qu'il y a une erreur aussi dans cette nouvelle version, car il sait bien que s'il le fait, tu lui répondras immédiatement par un troisième modèle expliquant que cette 2 eme erreur minuscule est corrigée et que ce troisième modèle sera aussi faux que le quatrième, que le cinquième, que le sixième que tu es susceptible de lui envoyer s'il te réponds, et cela jusqu'à la fin de ta vie, sans jamais reconnaître que ton modèle est juste faux.

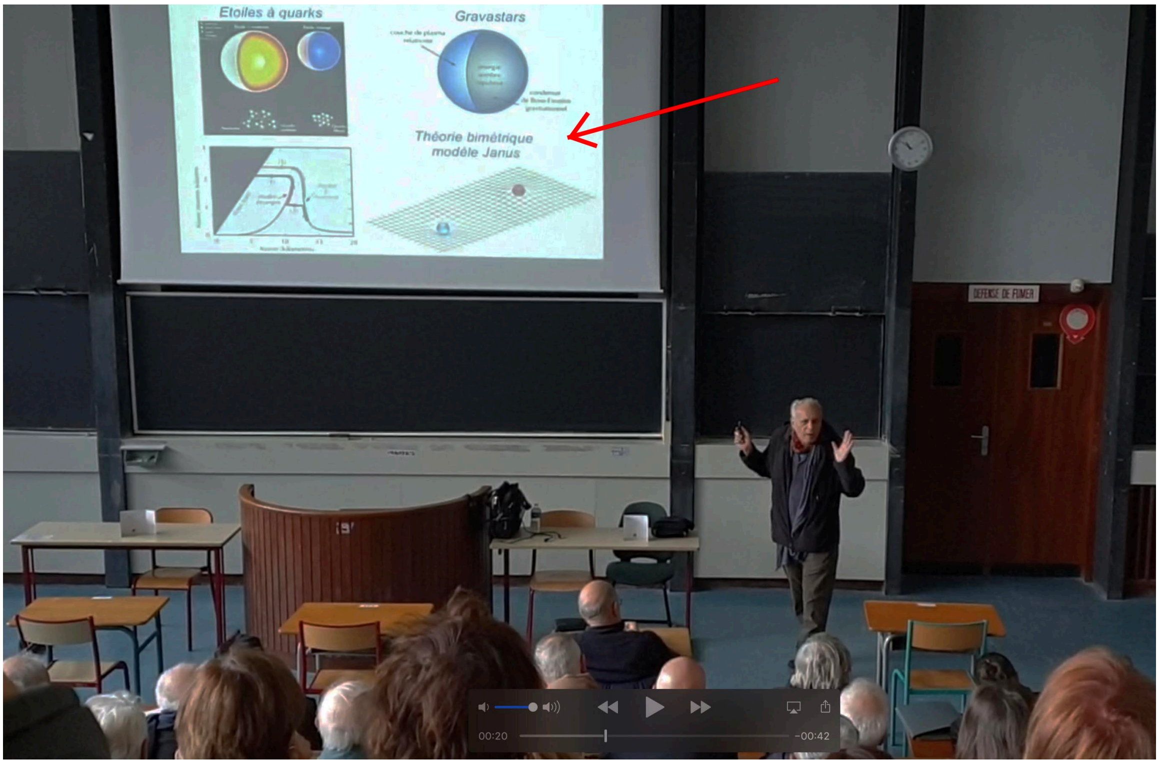
Je te confirme qu'il y n'a aucun doute pour moi (et pour plein de gens) que tu souffres de trouble mentaux

- **Je pense que c'est te rendre un immense service que de dire que tu es fou. De dire : Bien sûr que Jean-Pierre Petit est malade, mais bien qu'il soit fou ce qu'il dit mérite d'être pris en compte. Te présenter comme fou est le plus grand service qu'on puisse te rendre parce que c'est ce qui te permettra de passer la barrière". Je vais le dire à Luminet en lui envoyant la copie des lettres que je t'ai envoyé.**

Il écrit alors à Jean-Pierre Luminet en me mettant en copie de ce mails, où il met :

- **Tu connais aussi bien que moi le "cas Jean-Pierre Petit". Mon opinion (mais je serais intéressé d'avoir la tienne) c'est qu'il est capable d'être génial et/ou très intéressant sur certains points et que d'un autre il part souvent dans des délires logiques qui me paraissent proches (sans être aussi graves) que ceux de John Nash, mais je ne suis pas médecin.**

Comme mon ami Jean-Pierre Luminet se trouvait venir quelque jours plus tard à l'université Luminy de Marseille piour donner une conference je lui demande, pour prendre un peu le contrepied de mentionner mon modèle Janus, ce qu'il fait en incluant cette diapo dans sa présentation :



Ci-après la réaction de Luminet à l'évocation des mails de Staune :

Staune insiste en proposant de prendre à sa charge le déplacement d'un psychiatre de ses relations qui pourrait venir me voir et m'éclairer sur mon état :

Une personne atteinte de troubles mentaux graves peut produire des travaux scientifiques de qualité, John Nash est le meilleur exemple mais il y en plein d'autres

(le seul problème avec toi c'est qu'on ne sait pas encore si tu as produit des travaux de qualité **...** j'aimerais bien le savoir)

La seule et unique chose qui pourrait me faire rétracter que tu as des troubles mentaux cela c'est un examen sérieux par un psychiatre de qualité qui conclurait a ta parfaite normalité

Je suis prêt à financer cela. Que peux-tu demander de plus?

Staune Jean   staune@uip.edu

ANNEXE B : La première rédaction de cet article

The time independent spherically symmetric solution of the Einstein equation revisited

Jean-Pierre Petit, Gilles d'Agostini and Sebastien Michea +

Manaty Research group

Key words : Non contractible hypersurface. Throat sphere, space bridge. Spherically symmetric solution.

Abstract : Spherical symmetry does not immediately mean central symmetry. The time independent, spherically symmetric solution of the homogeneous Einstein equation is revisited with coordinates which keep the signature invariant and prevent time and radial coordinate interchange. The associated hypersurface is not contractible and corresponds to a space bridge linking two Minkowski spacetimes though a throat sphere. As the determinant of the metric vanishes on that sphere one gets an orbifold structure. When crossing that sphere the particles experience a PT, mass and energy inversions.

Introduction and main idea of this article.

In 1916 Karl Schwarzschild publishes [1] a solution of the vacuum Einstein equations (without second term) correspond to time translation invariance and spherical symmetry. It is only in 1999 that an English translation of this article will be available [2] thanks to S.Antoci et A.Loinger. Schwarzschild decides to express this solution by using a first set of real variables

(a) $$\{ t , x , y , z \} \in \mathbb{R}^4$$

The solution is then given in the form :

(b) $$ds^2 = F\,dt^2 - G\,(dx^2 + dy^2 + dz^2) - H\,(xdx + ydy + zdz)^2$$

He then introduces an intermediary variable :

(c) $$r = \sqrt{x^2 + y^2 + z^2}$$

Which, given (a) is essentially positive.

He performs a new coordinates change that allows him to simply express the spherical symmetry hypothesis :

(d) $$x = r \sin\theta \cos\varphi \qquad y = r \sin\theta \sin\varphi \qquad z = r \cos\theta$$

He then obains the form :

(e) $$ds^2 = F\,dt^2 - (G + H\,r^2)\,dr^2 - G\,r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)$$

In order for the metric to be lorentzian at infinity it is necessary that :

(f) $$r \to \infty \quad \text{implies} \quad F \to 1,\ G \to 1\ ,\ H \to 0$$

He introduces next a new change of coordinates :

(g) $$x_1 = \frac{r^3}{3} \quad , \quad x_2 = -\cos\theta \quad , \quad x_3 = \varphi \quad , \quad x_4 = t$$

Note that (a) + (c) imply that $x_1 \geq 0$

In those new coordinates the metric becomes :

(h) $$ds^2 = f_4\,dx_4{}^2 - f_1\,dx_1{}^2 - f_2\,\frac{dx_2{}^2}{1 - x_2{}^2} - f_3\,dx_3{}^2(1 - x_2{}^2)$$

where f_1 , $f_2 = f_3$, f_4 are three functions of x_1 (hence of r) that must satisfy the conditions :

- For large x_1 : $f_1 = \frac{1}{r^4} = (3x_1)^{-4/3}$, $f_2 = f_3 = r^2 = (3x_1)^{2/3}$, $f_4 = 1$

- The determinant $f_1\,f_2\,f_3\,f_4 = 1$

- The metric must be a solution of the field equation.

- Except for $x_1 = 0$ the f functions must be continuous.

His computation leads him to :

(i) $$f_1 = \frac{(3x_1 + \alpha^3)^{-4/3}}{1 - \alpha(3x_1 + \alpha^3)^{-1/3}} \qquad f_2 = f_3 = (3x_1 + \alpha^3)^{2/3} \qquad f_4 = 1 - 1 - \alpha(3x_1 + \alpha^3)^{-1/3}$$

α being an integration constant.

Using (g) we can rewrite his solution as :

(j) $$ds^2 = \left[1 - \frac{\alpha}{(r^3 + \alpha^3)^{1/3}}\right]dt^2 - \frac{r^4}{(r^3 + \alpha^3)\left[(r^3 + \alpha^3)^{1/3} - \alpha\right]}dr^2 - (r^3 + \alpha^3)^{2/3}(d\theta^2 + \sin^2\theta\,d\varphi^2)$$

That we can rewrite :

(k) $$ds^2 = g_{tt}\, dt^2 - g_{rr}\, dr^2 - g_{\theta\theta}\, d\theta^2 - g_{\varphi\varphi}\, d\varphi^2$$

The coefficients g_{tt} , g_{rr} et $g_{\theta\theta}$ are functions of the only variable r, the quantity $g_{\varphi\varphi}$ being a function of both r and φ . The variable r varies from 0 to infinity while being strictly positive by definition (c).

Let us limit ourself to the case $\alpha > 0$ and let consider a path corresponding to dt = dr = 0 . It has a perimeter :

(l) $$p = 2\pi (r^3 + \alpha^3)^{1/3}$$

This perimeter has a minimal value $p = 2\pi\alpha$.

The hypersurface solution is therefore non contractible.

The r variable cannot be considered as a « radial distance ». This hypersurface does not have a « centre », the object corresponding to r = 0 hence to à x = y = z = 0 is not a dot but a sphere of radius α .

We will now replicate Scharzschild's calculations by replacing the x, y, z and t variables by Greek characters (ρ is however kept for a later purpose), so that the reader not be tempted to assimilate them to distances (x, y and z) or time (t) and lose sight of what they
really are: real numbers.

Revisiting Schwarzchild computation.

Let us consider the zero second member Einstein equation $R_{\mu\nu} = 0$ in time independent and spherically symmetrical conditions. Let ξ_1 , ξ_2 , ξ_3 stand for rectangular coordinates, and ξ_o as the time marker, with $(\xi_0 , \xi_1 , \xi_2 , \xi_3) \in \mathbf{R}^4$ which stands real values for all coordinates. In addition we assume that there are no crossed terms in the line element, so that this last can be written :

(1) $$ds^2 = F d\xi_o^2 - G (d\xi_1^2 + d\xi_2^2 + d\xi_3^2) - H\, (\xi_1 d\xi_1 + \xi_2 d\xi_2 + \xi_3 d\xi_3)^2$$

where F , G , H are functions of $\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$.
At infinity we must have

(2) $$F \text{ and } G \rightarrow 1 \qquad H \rightarrow 0$$

Introduce the following coordinate change :

(3) $$\zeta = \sqrt{\xi_1^{\,2} + \xi_2^{\,2} + \xi_3^{\,2}}$$

(4) $$\theta = \mathrm{ar}\cos\left(\frac{\xi_3}{\zeta}\right)$$

(5) $$\varphi = \mathrm{arc}\cos\left(\frac{\xi_1}{\sqrt{\xi_1^{\,2} + \xi_2^{\,2}}}\right)$$

which goes with : $\zeta \in \mathbf{R}+ \quad \theta \in \mathbf{R} \quad \varphi \in \mathbf{R}$ and :

$$\xi_1 = \zeta \sin\theta \cos\varphi \;,\; \xi_2 = \zeta \sin\theta \sin\varphi \;,\; \xi_3 = \zeta \cos\theta$$

that we will call « pseudo spherical coordinates ». It gives :

(6) $$\mathrm{ds}^2 = \mathrm{F}\,\mathrm{d}\xi_o^{\,2} - (\,\mathrm{G} + \mathrm{H}\zeta^2\,)\,\mathrm{d}\zeta^{\,2} - \mathrm{G}\,\zeta^{\,2}(\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\,)$$

Introduce the following additional coordinate change : :

(7) $$\eta_1 = \frac{\zeta^{\,3}}{3} \quad , \quad \eta_2 = -\cos\theta \quad , \quad \eta_3 = \varphi$$

Then we have the volume element $\zeta^{\,2}\sin\theta\; d\zeta\; d\theta d\varphi \;=\; d\xi_1\, d\xi_2\, d\xi_3$. The new variables are then pseudo polar coordinate with the determinant 1. They have the evident advantage of polar coordinates for the treatment of the problem.

In the new pseudo polar coordinates :

(8) $$\mathrm{ds}^2 = \mathrm{F}\,\mathrm{d}\xi_o^{\,2} - (\frac{\mathrm{G}}{\zeta^4} + \frac{\mathrm{H}}{\zeta^2})\mathrm{d}\eta_1^{\,2} - \mathrm{G}\,\zeta^2\left[\frac{\mathrm{d}\eta_2^{\,2}}{1-\eta_2^{\,2}} + \mathrm{d}\eta_3^{\,2}\left(\,1-\eta_2^{\,2}\,\right)\right]$$

for which we write :

(9) $$ds^2 = f_4\, d\xi_o^{\,2} - f_1\, d\eta_1^{\,2} - f_2\,\frac{d\eta_2^{\,2}}{1-\eta_2^{\,2}} - f_3\, d\eta_3^{\,2}\,(1-\eta_2^{\,2})$$

Then $f_1 , f_2 = f_3 , f_4$ are functions of η_1 which have to fullfill the following conditions :

1 – For $\xi_1 = \infty$: $f_1 = 1/\zeta^4 = (3\eta_1)^{-4/3}$, $f_2 = f_3 = \zeta^2 = (3\eta_1)^{2/3}$, $f_4 = 1$

2 – The equation of the determinant : $f_1 \cdot f_2 \cdot f_3 \cdot f_4 = 1$

3 – The field equations.

4 – Continuity of the f , except for $\eta_1 = 0$

In order to formulate the field equations one must first form the components of the gravitational field corresponding to the line element (9). This happens in the simplest way when one builds the differential equation of the geodesic line by direct execution of the variation, and reads out the components of these. The differential equations of the geodesic line for the line element (9) immediatly result from the variation in the form :

(10)

$$0 = f_1 \frac{d^2\eta_1}{ds^2} + \frac{1}{2}\frac{\partial f_4}{\partial \eta_1}\left(\frac{d\eta_4}{ds}\right)^2 + \frac{1}{2}\frac{\partial f_1}{\partial \eta_1}\left(\frac{d\eta_1}{ds}\right)^2 - \frac{1}{2}\frac{\partial f_2}{\partial \eta_1}\left[\frac{1}{1-\eta_2^2}\left(\frac{d\eta_2}{ds}\right)^2 + (1-\eta_2^2)\left(\frac{d\eta_3}{ds}\right)^2\right]$$

(11)

$$0 = \frac{f_2}{1-\eta_2^2}\frac{d^2\eta_2}{ds^2} + \frac{\partial f_2}{\partial \eta_1}\frac{1}{1-\eta_1^2}\frac{d\eta_1}{ds}\frac{d\eta_2}{ds} + \frac{f_2\eta_2}{(1-\eta_1^2)^2}\left(\frac{d\eta_2}{ds}\right)^2 + f_2\eta_2\left(\frac{d\eta_3}{ds}\right)^2$$

(12)

$$0 = f_2(1-\eta_2^2)\frac{d^2\eta_3}{ds^2} + \frac{\partial f_2}{\partial \eta_1}(1-\eta_2^2)\frac{d\eta_1}{ds}\frac{d\eta_3}{ds} - 2 f_2 \eta_2 \frac{d\eta_2}{ds}\frac{d\eta_3}{ds}$$

(13)

$$0 = f_4 \frac{d^2\eta_4}{ds^2} + \frac{\partial f_4}{\partial \eta_1}\frac{d\eta_1}{ds}\frac{d\eta_4}{ds}$$

The comparison with

(14)

$$\frac{d^2\eta_\alpha}{ds^2} = -\frac{1}{2}\sum_{\mu,\nu}\Gamma^{\alpha}_{\mu\nu}\frac{d\eta_\mu}{ds}\frac{d\eta_\nu}{ds}$$

gives the components of the gravitational field :

(15a) $$\Gamma^{1}_{11} = -\frac{1}{2}\frac{\partial f_1}{\partial \eta_1}$$

(15b) $$\Gamma^{1}_{22} = +\frac{1}{2}\frac{1}{f_1}\frac{\partial f_2}{\partial \eta_1}\frac{1}{1-\eta_2^2}$$

(15c) $$\Gamma^{1}_{3\,3} = +\frac{1}{2}\,\frac{1}{f_1}\frac{\partial f_2}{\partial \eta_1}(1-\eta_2^{\,2})$$

(15d) $$\Gamma^{2}_{2\,1} = -\frac{1}{2}\,\frac{1}{f_2}\frac{\partial f_2}{\partial \eta_1}$$

(15e) $$\Gamma^{2}_{2\,2} = -\frac{\eta_2}{1-\eta_2^{\,2}}$$

(15f) $$\Gamma^{2}_{3\,3} = -\eta_2\,(1-\eta_2^{\,2})$$

(15g) $$\Gamma^{1}_{3\,1} = -\frac{1}{2}\,\frac{1}{f_2}\frac{\partial f_2}{\partial \eta_1}$$

(15h) $$\Gamma^{2}_{3\,2} = \frac{\eta_2}{1-\eta_2^{\,2}}$$

(15i) $$\Gamma^{4}_{4\,1} = -\frac{1}{2}\,\frac{1}{f_4}\frac{\partial f_4}{\partial \eta_1}$$

The other ones are zero. Due to rotational symmetry it is sufficient to write the field equations only for the equator $(\eta_2 = 0)$, therefore, since they will be differentiated only once, in the previous expressions it is possible to set everywhere since the begining $1-\eta_2^{\,2} = 0$. Then the calculation of the field equation gives :

(16a) $$\frac{\partial}{\partial \eta_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial \eta_1}\right) = \frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial \eta_1}\right)^2 + \left(\frac{1}{f_2}\frac{\partial f_2}{\partial \eta_1}\right)^2 + \left(\frac{1}{f_4}\frac{\partial f_4}{\partial \eta_1}\right)^2$$

(16b) $$\frac{\partial}{\partial \eta_1}\left(\frac{1}{f_1}\frac{\partial f_2}{\partial \eta_1}\right) = 2 + \frac{1}{f_1\,f_2}\left(\frac{\partial f_2}{\partial \eta_1}\right)^2$$

(16c) $$\frac{\partial}{\partial \eta_1}\left(\frac{1}{f_1}\frac{\partial f_4}{\partial \eta_1}\right) = \frac{1}{f_1\,f_4}\left(\frac{\partial f_4}{\partial \eta_1}\right)^2$$

Besides these three equations the functions f_1 , f_2 , f_3 must fullfill the equation of the determinant :

(17) $$f_1\,f_2^{\,2}\,f_4 = 1 \qquad \textit{i.e.} \quad \frac{1}{f_1}\frac{\partial f_1}{\partial \eta_1} + \frac{2}{f_2}\frac{\partial f_2}{\partial \eta_1} + \frac{1}{f_4}\frac{\partial f_4}{\partial \eta_1} = 0$$

For now we neglect (16b) and determine the three functions f_1 , f_2 , f_4 from (16a) , (16c) and (13). The equation (16c) can be transposed into the form :

(18) $$\frac{\partial}{\partial\eta_1}\left(\frac{1}{f_4}\frac{\partial f_4}{\partial\eta_1}\right)=\frac{1}{f_1 f_4}\frac{\partial f_1}{\partial\eta_1}\frac{\partial f_4}{\partial\eta_1}$$

This can be integrated and gives

(19) $$\frac{1}{f_4}\frac{\partial f_4}{\partial\eta_1}=\alpha\, f_1 \quad (\ \alpha \text{ being an integration constant })$$

The addition of (12a) and (12c') gives :

(20) $$\frac{\partial}{\partial\eta_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial\eta_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial\eta_1}\right)=\left(\frac{1}{f_2}\frac{\partial f_2}{\partial\eta_1}\right)^2+\frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial\eta_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial\eta_1}\right)^2$$

By taking (17) into account we get :

(21) $$-2\,\frac{\partial}{\partial\eta_1}\left(\frac{1}{f_2}\frac{\partial f_2}{\partial\eta_1}\right)=3\left(\frac{1}{f_2}\frac{\partial f_2}{\partial\eta_1}\right)^2$$

By integrating :

(22) $$\frac{1}{\dfrac{1}{f_2}\dfrac{\partial f_2}{\partial\eta_1}}=\frac{3}{2}\eta_1+\frac{\sigma}{2} \quad (\ \sigma \text{ integration constant })$$

or :

(23) $$\frac{1}{f_2}\frac{\partial f_2}{\partial\eta_1}=\frac{1}{3\eta_1+\sigma}$$

After a second integration :

(24) $$f_2=\lambda\,(\,3\eta_1+\sigma\,)^{2/3} \quad (\ \lambda \text{ integration constant })$$

The condition at infinity requires $\lambda = 1$. Then

(19) $$f_2=(\,3\eta_1+\sigma\,)^{2/3}$$

Hence it results further from (22) and (17)

(20) $$\frac{\partial f_4}{\partial\eta_1}=\alpha\, f_1 f_4=\frac{\alpha}{f_2^{\,2}}=\frac{\alpha}{(3\eta_1+\sigma)^{4/3}}$$

By integrating, while taking account the condition at infinity

(25) $$f_4 = 1-\alpha(3\eta_1+\sigma)^{-1/3}$$

Hence, from (17)

(26) $$f_1 = \frac{(3\eta_1+\sigma)^{-4/3}}{1-\alpha(3\eta_1+\sigma)^{-1/3}}$$

As it can be easily verified the equation (16b) is automatically fullfilled by the expression that we found for f_1 and f_2

Therefore all the conditions are satisfied except the *condition of continuity*.

f_1 will be discontinuous when $1=\alpha(3\eta_1+\alpha^3)^{-1/3}$, $3\eta_1=\alpha^3-\sigma$

In order this disconinuity coincides with the origin, it must be :

(27) $$\sigma=\alpha^3$$

Therefore the condition of continuity relates in this way the two integration constants σ and α . We have :

(28) $$3\eta_1+\alpha=\zeta^3+\alpha$$

(29) $$\mathrm{f}_4=1-\frac{\alpha}{(\zeta^3+\alpha^3)^{1/3}}$$

(30) $$\mathrm{f}_2=(\zeta^3+\alpha^3)^{2/3}$$

(31) $$\frac{\mathrm{d}\eta_2^2}{1-\eta_2^2}=\mathrm{d}\theta^2$$

Finally :

(32)

$$ds^2=\left(1-\frac{\alpha}{(\zeta^3+\alpha^3)^{1/3}}\right)d\xi_o^2-\frac{\zeta^4}{\left(\alpha^3+\zeta^3\right)\left[\left(\alpha^3+\zeta^3\right)^{1/3}-\alpha\right]}d\zeta^2-\left(\alpha^3+\zeta^3\right)^{2/3}\left(d\theta^2+\sin^2\theta\,d\phi^2\right)$$

When $\zeta\to\infty$ the line element tends to Lorentz form.

g_{tt} tends to zero when ζ tends to zero.

When $\zeta\to 0$ $\mathrm{g}_{\zeta\zeta}\simeq\frac{0}{0}$. An expansion into a series shows that $\mathrm{g}_{\zeta\zeta}\simeq\frac{3\zeta}{\alpha}\to 0$

Consider a loop located in the plane $\theta = 0$, with $d\xi_0 = 0$. It is no contractile : for ζ = 0 the perimeter of the loop tends to $2\pi\alpha$.

Notice that $\zeta = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$ is definitely not a « radius ». The point ζ = 0 does not correspond to some « center of symmetry ». The spherically symmetric requirement does not identify automatically to a central symmetry, as suggested by David Hilbert [5] [1] ζ is just one of the « space markers », nothing else. It's a number, not a length. The only length to be considered is the quantity s .

Expressing the metric in a better coordinate system.

Let us consider the coordinate system introduced in [3].

Introduce the new space marker ρ through the following coordinate change :

(33)
$$\zeta = |\alpha| \left[\left(1 + \mathrm{Log}\cosh\rho \right)^3 - 1 \right]^{1/3}$$

(30)
$$\rho = \pm \, \mathrm{argch} \left(e^{\sqrt[3]{1 + \left(\frac{\zeta}{|\alpha|}\right)^3} - 1} \right)$$

$$\zeta = 0 \quad \rightarrow \quad \rho = 0$$

With $\xi_o = c\,t$ the line element becomes :

(34)
$$ds^2 = \frac{\mathrm{Log}\cosh\rho}{1 + \mathrm{Log}\cosh\rho} c^2 dt^2 - \frac{1 + \mathrm{Log}\cosh\rho}{\mathrm{Log}\cosh\rho} \alpha^2 \tanh^2\rho \, d\rho^2 - \alpha^2 (1 + \mathrm{Log}\cosh\rho)^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

When $\rho \rightarrow \pm\infty$ we get the following Lorentz form :

(35)
$$ds^2 = c^2 dt^2 - \alpha^2 d\rho^2 - \alpha^2 \rho^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

We can figure this geometrical objet as a space bridge linking two Minkowski spacetimes, though a throat sphere whose perimeter is $2\pi\alpha$. We cannot think about its « radius » because that sphere has no center.

When $\rho \rightarrow 0$

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[1] We quote, page 67 of the german edition : « Die Gravitation $g_{\mu\nu}$ ist zentrisch symmetrisch in Bezug auf den Kooridinatenanfangspunkt. «

English translation : « The gravitation $g_{\mu\nu}$ is centrally symmetric with respect to the origin of coordinates. »

(36)
$$g_{tt} = \frac{\operatorname{Log}\cos h\,\rho}{1 + \operatorname{Log}\cos h\,\rho} c^2 \to 0$$

(37)
$$g_{\rho\rho} = -\frac{1 + \operatorname{Log}\cos h\,\rho}{\operatorname{Log}\cos h\rho}\ \alpha^2 \tan h^2 \rho\ \to \frac{0}{0}$$

(38)
$$g_{\theta\theta} \to \alpha^2 \qquad g_{\varphi\varphi} \to \alpha^2 \sin^2\theta$$

(39)
$$g_{\varphi\varphi} = \alpha^2 \sin^2\theta$$

We may easily overcome the indetermination (33) through an expansion into a series, which shows that when $\rho \to 0$

(40)
$$g_{\rho\rho} = -\frac{1 + \operatorname{Log}\cos h\,\rho}{\operatorname{Log}\cos h\,\rho}\ \alpha^2 \tan h^2 \rho\ \to\ -2$$

The determinant is :

(41)
$$\det g_{\mu\nu} = -\, c^2 \alpha^6 \tan h^2 \rho \sin^2\theta$$

It vanishes on the throat sphere. As a consequence, on this last we cannot define gaussian coordinates, so that the object is no longer a manifold but an orbifold. On the throat sphere the arrow of time and the space orientation cannot ne defined. This can be interpreted as a geometric structure where space and time are reversed through the throat sphere : when a particle crosses the throat sphere it experiences a PT-symmetry. According to Souriau's theorem [4] this T-inversion goes with a mass inversion.

In a future paper the physical interpretation of such solution will be investigated.

References :

[1] K. Schwarzschild : Über das Gravitionalsfeld einer Kugel Aus incompressibler Flüssigkeit nach der Einsteinschen Theorie. Sitzung der phys. Math. Klasse v.23 märz 1916

[2] K. Schwarzschild : On the gravitational field of a sphere of incompressible fluid according to Einstein theory. Translation by S.Antoci and A.Loinger. arXiv :physics/9905030v1 [physics.hist-ph] 12 may 1999.

[3] J.P.Petit & G.D'Agostini : Cancellation of the singularity of the Schwarzschild solution with natural mass inversion process. Mod. Phys. Lett. A vol. 30 n°9 2015

[4] J.M.Souriau : Structure des systèmes dynamiques. Dunod Ed. France, 1970 and Structure of Dynamical Systems. Boston, Birkhaüser Ed. 1997. For time inversion see page 190 equation (14.67).

[5] D.Hilbert : “Die Grundlagen der Physij. (Zweite Mitteilung)” in Nachchrichten von der Königlichen Gesellschaft zu Göttingen. Math. -Phys. Klasse. 1917 , p.53-76. Presented in the session of 23 december 1916

2019 : Première soumission à Physical Review D.

Rejet immédiat

Dear Jean-Pierre Petit, The title of this manuscript has two possible meanings:	Le titre de ce manuscrit a deux significations possibles :
either (1) that black holes do not exist as mathematical solutions to Einstein's equations,	soit (1) que les trous noirs n'existent pas en tant que solutions mathématiques aux équations d'Einstein,
or (2) that while they may exist as mathematical objects, actual black holes do not exist in the universe.	ou (2) que, bien qu'ils puissent exister en tant qu'objets mathématiques, les trous noirs réels n'existent pas dans l'univers.
Your manuscript is far short of establishing the first of these.	Votre manuscrit est loin d'établir la première d'entre elles.
The fully geodesically extended Schwarzschild solution has been studied in detail and is well understood.	La solution de Schwarzschild, entièrement étendue géodésiquement, a été étudiée en détail et est bien comprise. Nous contestons cette « extension » de la solution de Schwarzschild et le fait qu'elle ait été « bien comprise ».
Among its other properties, it has been shown that a classical particle cannot cross the Einstein-Rosen bridge from one exterior region to the other, contrary to what you appear to be saying on pages 4-6.	Parmi ses autres propriétés, il a été démontré qu'une particule classique ne peut pas traverser le pont Einstein-Rosen d'une région extérieure à l'autre, contrairement à ce que vous semblez dire aux pages 4-6.
Your various changes of variables add nothing that is correct to our knowledge of these solutions and their topology.	Vos différents changements de variables n'ajoutent rien de correct à notre connaissance de ces solutions et de leur topologie. Faux. Notre changement de variable permet d'établir un recollement de ce wormhole avec deux espaces « plats », décrits par des métriques de Lorentz, ce que ne fait pas l'interprétation d'Einstein-Rosen.

The last two sections of your manuscript (pages 8-9) relate to the second meaning of your title. Here you allude to a ``mass control'' mechanism that prevents neutron stars from reaching the critical mass and prevents the supermassive objects at the centers of galaxies from becoming black holes. The discussion there is quite brief and lacks the detailed and quantitative analysis that one would require tosupport such claims.	Les deux dernières sections de votre manuscrit (pages 8-9) concernent le second sens de votre titre. Vous faites ici allusion à un mécanisme de "contrôle de masse" qui empêche les étoiles à neutrons d'atteindre la masse critique et qui empêche les objets supermassifs au centre des galaxies de devenir des trous noirs. La discussion est assez brève et ne comporte pas l'analyse détaillée et quantitative qu'il faudrait pour étayer de telles affirmations. Ca n'est qu'une conjectiure en l'état, mais cela mérite d'être mentionné.
I regret to inform you that, in view of the above, we cannot accept your manuscript for publication in Physical Review D. Yours sincerely, Erick J. Weinberg Editor Physical Review D	J'ai le regret de vous informer que, compte tenu de ce qui précède, nous ne pouvons accepter votre manuscrit pour publication dans Physical Review D. Je vous prie d'agréer, Monsieur le Président, l'expression de mes sentiments distingués, Erick J. Weinberg Rédacteur en chef Examen physique D

4 décembre 2019. Rejet de la soumission à Physical Letters A.

Dear Dr. Michea, Physics Letters A is a multidisciplinary physics journal of letters. Correspondingly, we screen all incoming submission to determine whether they are suitable for publication in the format of an urgent letter. A number of different criteria are considered at this stage, which include timeliness, importance, accessibility, interest for a general physics audience and brevity of exposition. I read your manuscript with interest but I found it incremental in nature and not satisfying the criteria above. Based on this considerations, I regret to inform you that I made the decision not to publish your manuscript in Physics Letters A. Sincerely, Matteo Paris Editor Physics Letters A	Cher Docteur Michea Physical Letters A est un journal multidisciplinaire de physique publiant des lettres. Dans ces conditions nous examinons toute soumission pour déterminer si celles-ci conviennent ou non dans le format de publication d'une lettre urgente. Un certain nombre de crotères interviennent à ce stade qui incluent la rapidité, l'importance, l'accessibilité et l'intérêt pour un public intéressé par la physique en général. Est pris en compte également la concision. J'ai lu votre manuscrit avec intérêt, mais je l'ai trouvé de nature progressive (au sens d'une compilation) et ne répondant pas aux critères ci-dessus. Sur la base de ces considérations, j'ai le regret de vous informer que j'ai pris la décision de ne pas publier votre manuscrit dans les Lettres de Physique A. Sincerement vôtre, Matteo Paris Editor Physics Letters A
	Effectivement le papier n'était pas dans le format d'une lettre pour Physical Letters A. A réessayer avec la forme la plus concise possible.

On envoie alors à Modern Physics Letters A

La revue a du mal à trouver un refere :

8 janvier 2020. Dear Sebastien Michea, Thank you for your letter. Your manuscript is now under review with 2 pending reviews due on 19 and 23 Jan 2020. I apologise for any delays. Previous reviewer invitations have been met with declinations or unresponsiveness, and new invitations were made in their place. We seek your kind understanding on this. Kind regards, MPLA Editors WSPC Journal Office Modern Physics Letters A	8 janvier 2020. Cher Sébastien Michea, Merci pour votre lettre. Votre manuscrit est maintenant en cours d'évaluation, avec deux évaluations en attente qui doivent être effectuées les 19 et 23 janvier 2020. Je vous prie de m'excuser pour tout retard. Les précédentes invitations envers des examinateurs ont été refusées ou n'ont pas été prises en compte, et de nouvelles invitations ont été faites à leur place. Nous vous demandons de bien vouloir nous en excuser.

Referee report on the manuscript MPLA-D-19-00618	**Rapport du referee de Modern Physics LetterA. Manuscrit** D-19-0619
The authors analyse the case of spherically symmetric solutions of Einstein's equations in vacuum and discuss a bridgelike configuration.	Les auteurs analysent le cas d'une solution à symétrie sphérique d'une solution de l'équation d'Einstein dans le vide et discutent le cas d'une configuration en forme de pont.
There are a few technical details along the lines of Schwarzschild's original derivation of the schw solution, and a parallel method to obtain the metric solution being discussed.	Il y a quelques détails techniques sur la façon dont Karl Schwarzschild a construit sa solution. Une méthode parallèle est alors présentée pour construire la solution métrique, qui est discutée.
However, a number of points require clarifications, before this article may be considered for publication. These are elaborated below.	Quoi qu'il en soit un nombre de points demandent à être éclaircis avant qu'on puisse envisager une publication de cet article.
1. The authors claim that this solution is a new one. Schwarzschild solution is known to be the unique spherically symmetric vacuum solution to Einstein's equations (Brikhoff's theorem). How is this theorem evaded here to get a new solution?	1. Les auteurs disent qu'ils présentent une nouvelle solution. La solution de Schwarzschild est connue pour constituer la solution unique des équations d'Einstein dans le vide, en symétrie sphérique (théorème de Birkhoff). Comment peut on prétendre contourner ce théorème en proposant une nouvelle solution ? Nous connaissons le théorème de Birkhoff. Nous ne présentons pas « une nouvelle solution » mais la réinterprétation de cette solution (unique) construite par Schwarzschild, de l'équation d'Einstein en symétrie sphérique. Il va donc s'avérer nécessaire de bien mettre cela en lumière dans une nouvelle rédaction.
An interchange of space and time cordinates should lead to some solution that is coordinate equivalent to the interior Schwarzschild solution. Is this solution equivalent to that?	Une permutation entre une coordonnée d'espace et une coordonnée de temps devrait mener à une solution qui soit, du point de vue des coordonnées équivalente à la « solution intérieure de Schwarzschild ». Est-ce cela ?

	Nous supposons que le referee fait allusion à cette « construction » » d'une description de « l'intérieur de l'objet décrire par la métrique de Schwarzschild, qui passe (démarche classique) par le fait de dire que t devient une coordonnée radiale et r une coordonnée de temps. Non, ça n'a rien à voir. Il faudra bien le préciser dans une nouvelle rédaction.
2. The bridgelike solution from Schwarzschild may also be constructed, which is the well-known Einstein-Rosen bridge, where the throat is noncontractible. As per the argument in 1 above , this solution must be equivalent to ER bridge. Is it so? If not, then the same question has to be answered, that is, how is the Birkhoff's uniqueness theorem evaded ?	2.La solution en forme de pont, qui peut être construite à partir de Schwarzschild est le pont bien connu d'Einstein-Rosen. De même que dans l'argument 1 cette solution doit être équivalente à ce pont d'Einstein Rosen. Si ça n'est pas le cas, on doit répondre à la même question : comment peut-on échapper à la contrainte d'unicité issue du théorème de Birkhoff ? La formulation de la métrique que nous proposons est évidemment totalement équivalente au pont d'Einstein-Rosen, à a différence près, ce qui constitue l'apport principal, que dans ce cas ce pont relie deux espaces « plats », de Minkowski, ce que ne fait pas la formulation ER.
3. It is necessary to discuss the properties of the metric obtained in eq. 33-36 in more detail, such as its geodesic completeness, properties of curvature tensor and scalar etc. It also has to be compared with the Schwarzschild at least, since that is how the paper motivates itself in the beginning. No such discussion has been provided.	3. Il est aussi nécessaire de discuter les propriétés de la métrique obtenue dans les équations 33-36 avec plus de détails concernant les géodésiques les propriétés concernant la courbure tensorielle et scalaire. Cela doit aussi être comparé pour le moins avec Schwarzschild. Une telle discussion est absente. On reste dans le malentendu. Le referee croit toujours qu'il s'agit d'une « nouvelle solution » alors que ça n'est rien d'autre que la réinterprétation de la solution de Schwarzschild. La continuité des géométriques a été examinée dans notre papier de 2015. On en fera mention.
4. Based on the analysis here, it is not really clear whether this solution is a new	4. En se basant sur notre analyse on ne sait pas clairement s'il s'agit d'une nouvelle

solution to Einstein's equation or a coordinate transform of some well-known solution.	solution de l'équation d'Einstein ou d'une solution bien connue, exprimée à l'aide d'un changement de coordonnée. Réponse immédiate : conformément au théorème de Birkhoff il ne s'agit évidemment pas d'une « nouvelle solution » puisque ce théorème incontournable montre qu'il n'y a qu'une unique solution. Il s'agit donc d'une réinterprétation de celle-ci à travers un changement de variable.
5. Also, the authors mention bridge geometry and degeneracy of the metric, but has not mentioned any relevant reference on this which is unacceptable. For instance, they should have acknowledged Einstein-Rosen's original work, Bengtsson's work on Schwarzschild, Tseytlin's work on degenerate tetrad, Kaul-Sengupta's work on Schwarzschild and degenerate tetrad, bengtsson on degenerate metric etc.. e.g. A. A. Tseytlin, J. Phys. A: Math. Gen.15, L105 (1982). R.K. Kaul and S. Sengupta,Phys. Rev. D93, 084026 (2016) I. Bengtsson, Class. Quantum Grav. 8 (1991) 1847-1858. R.K. Kaul and S. Sengupta,Phys. Rev. D96, 104011 (2017). I. Bengtsson, Int. J. Mod. Phys. A4 (1989) 5527; S. Sengupta,Phys. Rev. D96, 104031 (2017). I. Bengtsson and T. Jacobson, 14, 3109 (1997) I. Bengtsson, Class Quantum Grav. 7 (1990) I. Bengts-son, Class. Quantum Grav. 8 (1991) 1847-1858;	5.En outre les auteurs mentionnent cette géométrie en « pont » (en fait le « wormhole », le « trou de ver ») mais omettent de citer des référence se rapportant à cette configuration ce qui est inacceptable. Ils devraient par exemple faire mention du papier original d'Einstein et Rosen, du travail de Bengtsson sur Schwarzschild, de ceux de Tseytlin et de Kaul-Sengupta sur sa forme dégénérée, etc. Pas de problème, on citera dans une nouvelle rédaction. **Conclusion** : Visiblement le referee a fait un effort méritoire de lecture, vis à vis d'un papier très complexe, où l'unicité de la solution et son identification avec une reformulation de la géométrie d'Einstein-Rosen n'avait pas été suffisamment mise en lumière. Les référence citée sont celles d'Indiens qui sont ceux qui ont publié sur cette géométrie wormhole en s'attachant à la singularité intrinsèque de la métrique (déterminant nul sur la sphère de gorge). On peut imaginer que le referee pourrait être l'un des auteurs cités.

ANNEXE C L’article dans la seconde rédaction

The time independent spherically symmetric solution of the Einstein equation revisited

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Key words : Non contractible hypersurface. Throat sphere, wormhole. Spherically symmetric solution.

Abstract : Reviewing the well-known paper of Karl Schwarzschild, 1916, we show that what is called "the Schwarzschild solution" is nothing but the linearized version of his work, that he worked out, in order to fit the approximate solution published by Einstein few months ago in order to take account of the advance of Mercure' s perihelion. So the classical interpretation of such solution is nothing but its extension to a complex field, which goes with the corresponding change of the signature. In 1935 Einstein and Rosen introduced a simple coordinate change which expressed this solution into a wormhole geometry. We present another one, with the same topology, which is asymptotically Minkowskian. We show that the singularity of the metric on the throat surface goes with a PT symmetry of the two connected folds. Based on dynamic groups theory, when particles cross that border they experience mass an energy inversion.

I-INTRODUCTION AND MAIN IDEA OF THIS ARTICLE

In 1916 Karl Schwarzschild publishes [1] a solution of the vacuum Einstein equations (without second term) which exhibits time translation invariance and spherical symmetry. It is only in 1999 that an English translation of this article will be available [2] thank to S.Antoci and A.Loinger [2].
Birkhoff's theorem shows that this solution is unique so that what follows must not be considered as a "new solution of the Einstein's equation" which would contradict that theorem. There are just different interpretations of that unique solution, through different coordinate choices.

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- As shown by Einstein and Rosen [3] such coordinate changes go with topological option. In addition such choices suggest that the associated coordinates could be identified to physical quantities. I may always redefine the field when expressing these solutions which may not be evident for researchers. .

We first suggest to get into the history of such a solution.

In 1915 A. Einstein publishes a first approximate solution of his field equation, in order to take account of the advance of the perihelion of Mercury.

In 1916, aged 42 years, Karl Schwarzschild is assigned at the border of Russia after he enlisted in the first world war. Mathematician, he closely follows Einstein's works, and builds immediately an exact solution of his equation. His main objective is to converge towards the linearized solution of the latter. Indeed at that time nobody imagined that a non-linear solution could have a physical meaning. Thus he does not completely explain his non-linear solution, which we will do in what follows .

Both Einstein and Schwarzschild understood this approach as follows:

- An equation exists, expressed using tensors. Its solution is a metric, which can be expressed using an infinite number of sets of coordinates, which are in fact simple numbers. Hereafter is the equation that Einstein has just published.

feldes bezeichnen.

Ist in dem betrachteten Raume »Materie« vorhanden, so tritt deren Energietensor auf der rechten Seite von (2) bzw. (3) auf. Wir setzen

$$\boxed{G_{im} = -\varkappa\left(T_{im} - \frac{1}{2} g_{im} T\right),} \qquad (2\,a)$$

wobei

$$\sum_{\varrho\sigma} g^{\varrho\sigma} T_{\varrho\sigma} = \sum_{\sigma} T_{\sigma}^{\sigma} = T \qquad (5)$$

gesetzt ist; T ist der Skalar des Energietensors der »Materie«, die rechte

Fig.1 : The Einstein's equation as it appears in his first publication.

The solution is a metric from which the geodesics of a four-dimensional hypersurface can be calculated. Particles and photons are assumed to travel along these geodesics.

Einstein builds the first approximate solution of the equation without a second member and in the case of a solution with stationary spherical symmetry. The choice of this Riemannian metric with signature $(+\,-\,-\,-)$ accounts for this symmetry. In addition, that metric must describe an asymptotically flat space-time, corresponding to special relativity theory.

On this hypersurface the only physical quantity is the length s, which is a real quantity. The coordinates are just real numbers defined on $\mathbb{R}4$. We insist on that point. A priori the signature is an invariant. If not the definition of s may be changed and expressed in $\mathbb{C}$.

In 1916 the great mathematician David Hilbert published a long text entitled "The Foundations of Physics". At that time it was imagined that the only fields governing physics were gravitation and electromagnetism. In his essay Hilbert therefore undertakes to create an ambitious formalism that integrates these two phenomena. When he discovers the solution produced by Schwarzschild (who died of a forehead ailment shortly after publishing his work) he hurries to integrate it into a second version of his dissertation [4].

Hilbert is primarily a mathematician, not a physicist. Whereas a man like Einstein is first and foremost an excellent physicist who strives to use the mathematics of his time to account for his fantastic intuitions. Let's recall the anecdote where Hilbert was asked to give a lecture in front of engineers, replacing his colleague Klein, who was ill. The latter had started his talk by saying:

> - *It is often said that mathematicians and physicists have trouble understanding each other. This is not true. In fact, they have nothing to do with each other.*

In the rest of his life, he changed his mind by producing with R.Courant a landmark work entitled "Methods of mathematical physics". But before that Hilbert had a vision of the universe that is the one of an aesthete-mathematician. Reading his dissertation shows that he did not understand the deep meaning of Einstein's approach. For him, the fourth dimension is a pure imaginary quantity which is grafted onto a pre-existing three-dimensional space. Unlike Einstein, who had the genius to understand that these four dimensions were interchangeable and that time had to be measured in metres.

Die Grundlagen der Physik.

(Zweite Mitteilung.)

Von

David Hilbert.

Vorgelegt in der Sitzung vom 23. Dezember 1916.

In meiner ersten Mitteilung [1]) habe ich ein System von Grundgleichungen der Physik aufgestellt. Ehe ich mich zur Theorie der Integration dieser Gleichungen wende, erscheint es nötig, einige allgemeinere Fragen sowohl logischer wie physikalischer Natur zu erörtern.

Zunächst führen wir an Stelle der Weltparameter w_s ($s = 1, 2, 3, 4$) die allgemeinsten reellen Raum-Zeit-Koordinaten x_s ($s = 1, 2, 3, 4$) ein, indem wir

$$w_1 = x_1, \qquad w_2 = x_2, \qquad w_3 = x_3, \qquad w_4 = ix_4$$

Fig.2 : Hilbert's second paper. We note that he announces from the outset

that his fourth dimension (arrow) will be purely imaginary.

In this second version [4] Hilbert introduces the word "Zentrisch symmetrisch" which means "central symmetry", which presupposes that the space related to this solution could have a "center of symmetry". There is a fundamental difference between "central symmetry" and "spherical symmetry". Just as in the geometry of a two-dimensional torus we are faced with a "circular symmetry" and not an "axial symmetry"; simply because in this torus-shaped space there is no axis.

2. Die $g_{\mu\nu}$ sind von der Zeitkoordinate x_4 unabhängig.

3. Die Gravitation $g_{\mu\nu}$ ist zentrisch symmetrisch in Bezug auf den Koordinatenanfangspunkt.

Nach Schwarzschild ist die allgemeinste diesen Annahmen entsprechende Maßbestimmung in räumlichen Polarkoordinaten, wenn

Fig.3 : Hilbert 's central symmetry

Hilbert endows the hypersurface with polar coordinates ("Polarkoordinaten"). The angles reflect spherical symmetry, but the introduction of this "radial" coordinate r suggests that the object may have a "pole in r = 0. At this stage, he first introduces the fourth dimension in the form of an undefined quantity denoted by the letter l.

Nach Schwarzschild ist die allgemeinste diesen Annahmen entsprechende Maßbestimmung in räumlichen Polarkoordinaten, wenn

$$\begin{aligned} w_1 &= r\cos\vartheta \\ w_2 &= r\sin\vartheta\cos\varphi \\ w_3 &= r\sin\vartheta\sin\varphi \\ w_4 &= l \end{aligned}$$

gesetzt wird, durch den Ausdruck

$$F(r)\,dr^2 + G(r)\,(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) + H(r)\,dl^2 \tag{42}$$

Fig.4 : Hilbert's introduction of a fourth dimension noted l
"Polarkoodinaten": "polar coordinates".

Note in its expression (42) that it retains only the bilinear form and that the ds^2 is absent. In the rest of his text this fourth variable l is converted into it, which clearly expresses his conception of time: it is for him an imaginary quantity, which is therefore not of the same nature as the other three dimensions of such spacetime, which contradicts Einstein's vision.

keine wesentliche Einschränkung bedeutet, so ergibt sich aus (43) für $l = it$ die gesuchte Maßbestimmung in der von Schwarzschild zuerst gefundenen Gestalt

$$(45) \quad G(dr, d\vartheta, d\varphi, dl) = \frac{r}{r-\alpha} dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2 - \frac{r-\alpha}{r} dt^2.$$

Fig.5 : How after Hilbert's case, time becomes imaginary quantity

Here again we will notice that the ds^2 is absent and that Hilbert concentrates his attention exclusively on this bilinear form G. This will lead his successors to opt for a presentation of the line element according to :

$$(1) \qquad ds^2 = r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2 + \frac{1}{1-\frac{\alpha}{r}} dr^2 - (1 - \frac{\alpha}{r}) dt^2$$

associated with a signature (+ + + -), whereas Einstein, Schwarzschild, Weyl, and all the mathematicians who followed at that time this development of the Relativity theory use the signature (+ - - -). This expression has since been designated as "Schwarzschild's metric" and the letter r which appears in it is then identified with a "radial coordinate" likely to be given a null value, synonymous with the existence of a "central singularity".

Does this expression correspond to Schwarzschild's 1916 publication? The answer is no. To find out, we will go through his own text line by line. In the following, the Schwarzschild contribution is indicated in indented text to make it clear that this is not a personal contribution, a "new solution", nothing but an identical reproduction of Schwarzschild's article. We have also marked the equations in his article with letters, to distinguish this part from what will be indicated later.

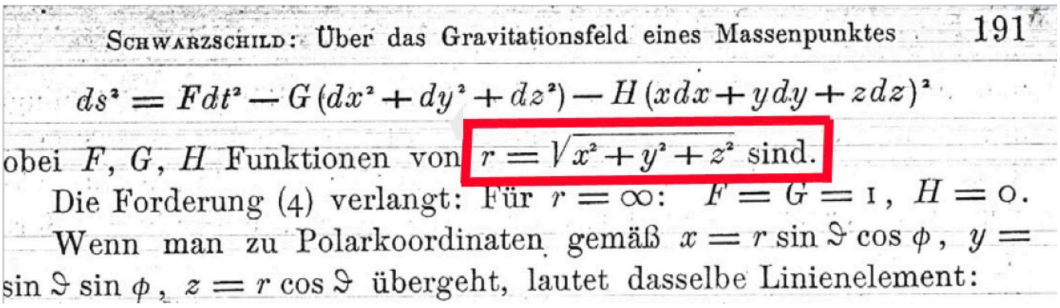
II THE SCHWARZSCHILD PAPER

He writes :

> Let us consider the zero second member Einstein equation $R_{\mu\nu} = 0$ in time independent and spherically symmetrical conditions. Let x, y, z stand for rectangular coordinates, and t as the time marker, with $\{ t, x, y, z \} \subset \mathbb{R}4$ which takes on real values for all coordinates. In addition, we assume that there are no crossed terms in the line element, so that this last can be written:
>
> $$(a) \qquad ds^2 = F dt^2 - G(dx^2 + dy^2 + dz^2) - H\,(x dx + y dy + z dz)^2$$
>
> where F, G, H are functions of $\sqrt{x^2 + y^2 + z^2}$.
>
> At infinite we must have

(b) $$F \text{ and } G \rightarrow 1 \qquad H \rightarrow 0$$

Introduce the following coordinate change: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$$ds^2 = Fdt^2 - G(dx^2+dy^2+dz^2) - H(xdx+ydy+zdz)^2$$

obei F, G, H Funktionen von $\boxed{r = \sqrt{x^2+y^2+z^2} \text{ sind.}}$

Die Forderung (4) verlangt: Für $r = \infty$: $F = G = 1$, $H = 0$.

Wenn man zu Polarkoordinaten gemäß $x = r \sin \vartheta \cos \phi$, $y =$ sin ϑ sin ϕ, $z = r \cos \vartheta$ übergeht, lautet dasselbe Linienelement:

Fig.6 : How Schwarzschild defines r

(c) $$\mathrm{r} = \sqrt{\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2} \geq 0$$

(d) $$\theta = \mathrm{ar}\cos\left(\frac{\mathrm{z}}{\mathrm{r}}\right)$$

(e) $$\varphi = \mathrm{arc}\cos\left(\frac{\mathrm{x}_1}{\sqrt{\mathrm{x}^2 + \mathrm{y}^2}}\right)$$

which goes with: $\mathrm{r} \in \mathbb{R}+ \quad \theta \in \mathbb{R} \quad \varphi \in \mathbb{R}$ and:

$$x = r \sin\theta \cos\varphi \ , \ y = r \sin\theta \sin\varphi \ , \ z = r \cos\theta$$

that we will call « pseudo spherical coordinates ». It gives:

(f) $$ds^2 = F\,dt^2 - (G + H\,r)\,dr^2 - G\,r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)$$

Introduce the following additional coordinate change:

(g) $$x_1 = \frac{r^3}{3} \qquad x_2 = -\cos\theta \qquad x_3 = \varphi$$

Note that these quantities x_1, x_2, x_3 have nothing to do with coordinates of a tridimensional space in the trivial sense. They are only auxiliary quantities, which Schwarzschild makes abundant use of in his article.

Then we have the volume element $r^2 \sin\theta \; dr\, d\theta d\varphi = dx\, dy\, dz$. The new variables are then pseudo polar coordinate with the determinant 1. They have the evident advantage of polar coordinates for the treatment of the problem.

In the new pseudo polar coordinates:

(h) $$ds^2 = F dt^2 - \left(\frac{G}{r^4} + \frac{H}{r^2}\right) dx_1^2 - G r^2 \left[\frac{dx_2^2}{1 - x_2^2} + dx_3^2 \left(1 - x_2^2\right) \right]$$

for which we write:

(i) $$ds^2 = f_4 dt^2 - f_1 dx_1^2 - f_2 \frac{dx_2^2}{1 - x_2^2} - f_3 dx_3^2 (1 - x_2^2)$$

Then f_1, $f_2 = f_3$, f_4 are functions of η_1 which have to fulfil the following conditions :

1 – For $x_1 = \infty$: $f_1 = 1/r^4 = (3x_1)^{-4/3}$, $f_2 = f_3 = r^2 = (3x_1)^{2/3}$, $f_4 = 1$

2 – The equation of the determinant: $f_1 \cdot f_2 \cdot f_3 \cdot f_4 = 1$

3 – The field equations

4 – Continuity of the f, except for $x_1 = 0$

In order to formulate the field equations, one must first form the components of the gravitational field corresponding to the line element (i). This happens in the simplest way when one builds the differential equation of the geodesic line by direct execution of the variation and reads out the components of these. The differential equations of the geodesic line for the line element (i) immediately result from the variation in the form:

(j)

$$0 = f_1 \frac{d^2 x_1}{ds^2} + \frac{1}{2} \frac{\partial f_4}{\partial x_1} \left(\frac{dx_4}{ds}\right)^2 + \frac{1}{2} \frac{\partial f_1}{\partial x_1} \left(\frac{dx_1}{ds}\right)^2 - \frac{1}{2} \frac{\partial f_2}{\partial x_1} \left[\frac{1}{1 - x_2^2} \left(\frac{dx_2}{ds}\right)^2 + (1 - x_2^2) \left(\frac{dx_3}{ds}\right)^2 \right]$$

(k)

$$0 = \frac{f_2}{1 - x_2^2} \frac{d^2 x_2}{ds^2} + \frac{\partial f_2}{\partial x_1} \frac{1}{1 - x_1^2} \frac{dx_1}{ds} \frac{dx_2}{ds} + \frac{f_2 x_2}{(1 - x_1^2)^2} \left(\frac{dx_2}{ds}\right)^2 + f_2 x_2 \left(\frac{dx_3}{ds}\right)^2$$

(l)

$$0 = f_2 (1 - x_2^2) \frac{d^2 x_3}{ds^2} + \frac{\partial f_2}{\partial x_1} (1 - x_2^2) \frac{dx_1}{ds} \frac{dx_3}{ds} - 2 f_2 x_2 \frac{dx_2}{ds} \frac{dx_3}{ds}$$

(m)

$$0 = f_4 \frac{d^2 x_4}{ds^2} + \frac{\partial f_4}{\partial x_1} \frac{dx_1}{ds} \frac{dx_4}{ds}$$

The comparison with

(n)

$$\frac{d^2 x_\alpha}{ds^2} = -\frac{1}{2} \sum_{\mu,\nu} \Gamma^{\alpha}_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds}$$

gives the components of the gravitational field:

(o-a) $$\Gamma^{1}_{11} = -\frac{1}{2} \frac{\partial f_1}{\partial x_1}$$

(o-b) $$\Gamma^{1}_{22} = +\frac{1}{2} \frac{1}{f_1} \frac{\partial f_2}{\partial x_1} \frac{1}{1 - x_2^2}$$

(o-c) $$\Gamma^{1}_{33} = +\frac{1}{2} \frac{1}{f_1} \frac{\partial f_2}{\partial x_1} (1 - x_2^2)$$

(o-d) $$\Gamma^{2}_{21} = -\frac{1}{2} \frac{1}{f_2} \frac{\partial f_2}{\partial x_1}$$

(O-e) $$\Gamma^{2}_{22} = -\frac{x_2}{1 - x_2^2}$$

(o-f) $$\Gamma^{2}_{33} = -x_2 (1 - x_2^2)$$

(o-g) $$\Gamma^{1}_{31} = -\frac{1}{2} \frac{1}{f_2} \frac{\partial f_2}{\partial x_1}$$

(o-h) $$\Gamma^{2}_{32} = \frac{x_2}{1 - x_2^2}$$

(o-i) $$\Gamma^{4}_{41} = -\frac{1}{2} \frac{1}{f_4} \frac{\partial f_4}{\partial x_1}$$

The other ones are zero. Due to rotational symmetry it is sufficient to write the field equations only for the equator ($x_2 = 0$), therefore, since they will be differentiated only once, in the previous expressions it is possible to set everywhere since the beginning $1-x_2^{\,2}=0$. Then the calculation of the field equation gives:

(p-a) $$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}\right)=\frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}\right)^2+\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2+\left(\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)^2$$

(p-b) $$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_2}{\partial x_1}\right)=2+\frac{1}{f_1 f_2}\left(\frac{\partial f_2}{\partial x_1}\right)^2$$

(p-c) $$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_4}{\partial x_1}\right)=\frac{1}{f_1 f_4}\left(\frac{\partial f_4}{\partial x_1}\right)^2$$

Besides these three equations the functions f_1 , f_2 , f_3 must fulfil the equation of the determinant:

(q) $$f_1 f_2^{\,2} f_4=1 \qquad i.e. \quad \frac{1}{f_1}\frac{\partial f_1}{\partial x_1}+\frac{2}{f_2}\frac{\partial f_2}{\partial x_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}=0$$

For now, we neglect (16b) and determine the three functions f_1 , f_2 , f_4 from (16a), (16c) and (13). The equation (16c) can be transposed into the form:

(r) $$\frac{\partial}{\partial x_1}\left(\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)=\frac{1}{f_1 f_4}\frac{\partial f_1}{\partial x_1}\frac{\partial f_4}{\partial x_1}$$

This can be integrated and gives

(s) $$\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}=\alpha f_1 \quad (\,\alpha \text{ being an integration constant})$$

The addition of (12a) and (12c') gives:

(t) $$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)=\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2+\frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)^2$$

By taking (17) into account we get:

(u) $$-2\frac{\partial}{\partial x_1}\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)=3\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2$$

By integrating:

(v) $$\frac{1}{\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}} = \frac{3}{2}x_1 + \frac{\rho}{2} \quad (\ \rho \text{ integration constant})$$

or :

(w) $$\frac{1}{f_2}\frac{\partial f_2}{\partial x_1} = \frac{1}{3x_1 + \rho}$$

After a second integration:

(x) $$f_2 = \lambda(3x_1 + \rho)^{2/3} \quad (\ \lambda \text{ integration constant})$$

The condition at infinity requires λ = 1. Then

(y) $$f_2 = (3x_1 + \rho)^{2/3}$$

Hence it results further from (22) and (17)

(z) $$\frac{\partial f_4}{\partial x_1} = \alpha f_1 f_4 = \frac{\alpha}{f_2^{\,2}} = \frac{\alpha}{(3x_1 + \rho)^{4/3}}$$

By integrating, while taking account the condition at infinity

(z-a) $$f_4 = 1 - \alpha(3x_1 + \rho)^{-1/3}$$

Hence, from (q)

(z-b) $$f_1 = \frac{(3x_1 + \rho)^{-4/3}}{1 - \alpha(3x_1 + \rho)^{-1/3}}$$

As it can be easily verified the equation (16b) is automatically fulfilled by the expression that we found for f_1 and f_2

Therefore, all the conditions are satisfied except the *condition of continuity*.

f_1 will be discontinuous when $\quad 1 = \alpha(3x_1 + \alpha^3)^{-1/3} \quad , \quad 3x_1 = \alpha^3 - \rho$

In order for this discontinuity to coincide with the origin, it must be:

(z-c) $$\rho = \alpha^3$$

Therefore the condition of continuity relates in this way the two integration constants ρ and α . The complete solution of our problem reads now :

In what precedes we have only reproduced, line after line, Schwarzschild's calculation, without the slightest modification. We are now going to add the lines of calculation that he should logically have included to express his exact solution, instead of branching off to Einstein's linearized solution.

III- SUMMARY OF THE SCHWARZSCHILD'S ARTICLE.

Karl Schwarzschild started from an M4 manifold with the four coordinates

$$(t \ , \ x \ , \ y \ , \ z) \in \mathbf{R}^4$$

Just keep in mind that these four letters are simple real numbers, a time marker t and three space markers x , y , z .

From there he introduces a first auxilliary quantity

$$r = \sqrt{x^2 + y^2 + z^2} \ \geq 0$$

which must not be identified to a "radial distance", otherwise a major mistake will be made. Here again it is a simple number, positive or nil by construction. Then he accounts for the symmetry by writing :

$$ds^2 = F dt^2 - G (dx^2 + dy^2 + dz^2) - H \ (x dx + y dy + z dz)^2$$

He then introduce two angles.

$$ds^2 = F \, dt^2 - (G + H r) dr^2 - G r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2)$$

At this stage he introduces three auxiliary quantities x_1 , x_2 , x_3 which have nothing to do with trirectangular coordinates and functions f_1 , f_2 , f_3, which leads him to write his line elements as :   :

$$ds^2 = f_4 dt^2 - f_1 dx_1^2 - f_2 \frac{dx_2^2}{1 - x_2^2} - f_3 dx_3^2 (1 - x_2^2)$$

He calculates the Christoffel symbols and then expresses these functions f_1 , f_2 , f_3 taking into account boundary conditions, introducing an integration constant α , the "Schwarzschild length".

By expressing its results using its intermediate quantity r, he gets :

(2) $$f_1 = \frac{1}{(r^3 + \alpha^3)^{4/3} \left[1 - \alpha (r^3 + \alpha^3)^{1/3} \right]}$$

(3) $$f_2 = f_3 = (r^3 + \alpha^3)^{2/3}$$

(4) $$f_4 = 1 - \alpha (r^3 + \alpha^3)^{-1/3}$$

This should logically have led him to express his solution as :

(5)

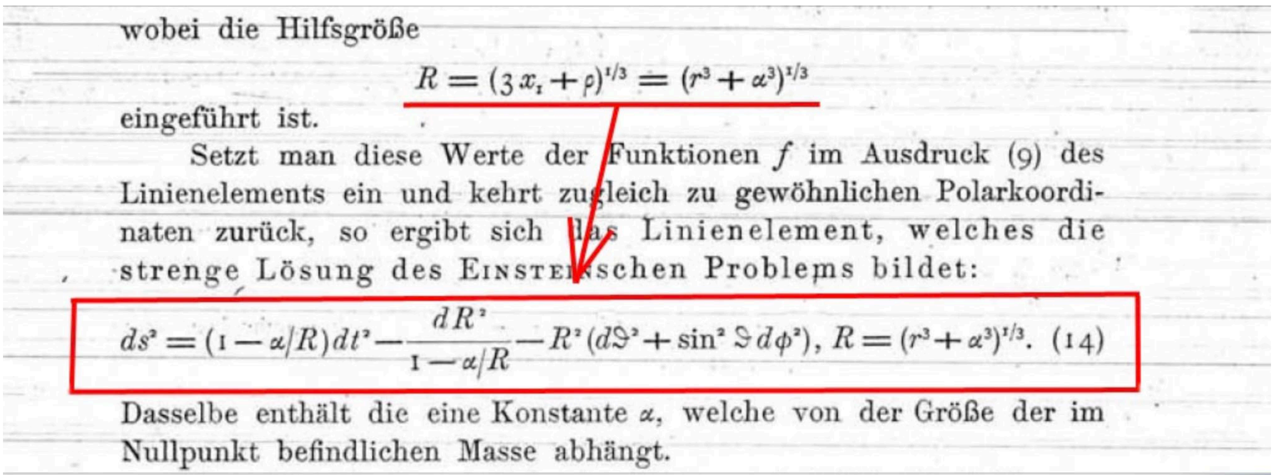
$$\boxed{ds^2 = \left[1-\alpha(r^3+\alpha^3)^{-1/3}\right]dt^2 - \frac{r^2 dr^2}{(r^3+\alpha^3)^{2/3}\left[1-\alpha(r^3+\alpha^3)^{-1/3}\right]} - (r^3+\alpha^3)^{2/3}(d\theta^2+\sin^2\theta\, d\varphi^2)}$$

This is just THE exact, non-linear solution of Schwarzschild, which he did not express, in which it would be totally wrong to assimilate r at a radial distance. This quantity is definitively positive. If we make r tend towards infinity the line element becomes that of a flat 4-space with hyperbolic signature $(+---)$ which corresponds to the physics of Special Relativity in a vacuum.

Now let's do an expansion into a series, considering $\frac{\alpha}{r}$ as a small quantity:

(6) $$ds^2 = \left(1-\frac{\alpha}{r}\right)dt^2 - \frac{dr^2}{1-\frac{\alpha}{r}} - r^2(d\theta^2+\sin^2\theta\, d\varphi^2)$$

which is fallaciously considered as the so-called "Schwarzschild's solution" and which is in fact only its linearized form when r is large compared to Schwarzschild's length α, which is the case for the Sun. Schwarzschild never considered the non-linear form since he was not able to refer to an astronomical phenomenon. In his article Schwarzschild obtains this expression (6) using a change of variable, which prevented scientists from realizing that it was only Einstein's linearized solution, corresponding to $r >> \alpha$.



wobei die Hilfsgröße

$$R = (3x_1 + \rho)^{1/3} = (r^3+\alpha^3)^{1/3}$$

eingeführt ist.

Setzt man diese Werte der Funktionen f im Ausdruck (9) des Linienelements ein und kehrt zugleich zu gewöhnlichen Polarkoordinaten zurück, so ergibt sich das Linienelement, welches die strenge Lösung des Einsteinschen Problems bildet:

$$\boxed{ds^2 = (1-\alpha/R)dt^2 - \frac{dR^2}{1-\alpha/R} - R^2(d\vartheta^2+\sin^2\vartheta\, d\phi^2), \quad R=(r^3+\alpha^3)^{1/3}. \quad (14)}$$

Dasselbe enthält die eine Konstante α, welche von der Größe der im Nullpunkt befindlichen Masse abhängt.

Fig.7 : The Schwarzschild auxilliary quantity R (« Hilfsgrößе »)

The words "wobei die Hilfsgröbe eingefüst ist" literally mean "where the auxiliary variable is introduced". For R is nothing but an auxiliary variable which cannot, by its very definition, given by Schwarzschild, take on a value less than α , less than the Schwarzschild length.

Unless one envisages going outside the initial defined framework $\{ t, x, y, z \} \subset \mathbb{R}4$ with r defined as $r = \sqrt{x^2 + y^2 + z^2} \geq 0$

It should be noted that when r tends towards zero the perimeter corresponding to a 2π variation of the variable φ tends towards $2\pi\alpha$ which gives this geometry a non-contractile aspect, which was immediately noticed by several researchers. At this stage the hypersurface can be assimilated to a bordered manifold.

Other Note : for r = 0 in expression (5) the terms g_{tt} and g_{rr} are zero, which introduces a double degeneration of the metric.

(7) $$d\sigma^2 = r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

If we wanted to consider regions of the hypersurface where R would be lower than α this would lead to negative r values which would contradict its definition. This point has been highlighted by Abrams [5] (shortly before his death) and by Antoci [6].

IV – THE EINSTEIN-ROSEN WORMHOLE

This was firstly introduced by L.Flamm in 1916 [7] , then by Einstein and Rosen in 1935 [8] through the following change of variable :

(8) $$r = \alpha + u^2 = 2m + u^2$$

leading to:

(9) $$ds^2 = \left[\frac{u^2}{u^2 + 2m} \right] dt^2 - 4 (u^2 + 2m) du^2 - (u^2 + 2m)^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

On the throat the metric tends to :

(10) $$ds^2 = \left[\frac{u^2}{2m} \right] dt^2 - 8m \, du^2 - 4m^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

- On the throat $g_{tt} = 0$ but not g_{uu} . The double degeneration of the metric is therefore not evident on that throat.

When u tends to infinity the line element becomes :

(11) $$ds^2 = dt^2 - 4u^2 du^2 - u^4 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

which does not identify to a flat metric.

In various studies ([10], [18]), a discontinuity-free connection between the geodesics of the two layers is noted.

V – THE PETIT'S WORMHOLE

Introduce the new coordinate change :

(12) $$\mathrm{r} = \alpha + \mathrm{Log\,ch}\rho$$

which gives :

(13)

$$\mathrm{ds}^2 = \frac{\mathrm{Log\,ch}\,\rho}{1+\mathrm{Log\,ch}\,\rho}\mathrm{c}^2\mathrm{dt}^2 - \frac{1+\mathrm{Log\,ch}\,\rho}{\mathrm{Log\,ch}\rho}\ \mathrm{th}^2\rho\,\mathrm{d}\rho^2 - (\alpha + \mathrm{Log\,ch}\,\rho)^2(\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\,)$$

When ρ tending towards infinity this metric tends towards the Lorentz metric of a flat space with hyperbolic signature $(+---)$.

(14) $$\mathrm{ds}^2 = \mathrm{c}^2\mathrm{dt}^2 - \mathrm{d}\rho^2 - \rho^2(\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\,)$$

On the throat $g_{tt} = 0$ and the term $g_{\rho\rho}$ gets an indeterminate form $\frac{0}{0}$. In the vicinity to the throat a development into a series of the functions $ch\,\rho$ and $sh\rho$ shows $g_{\rho\rho}$ that tends to 2. Here again we get a wormhole structure. The degenerated metric is :

(15) $$\mathrm{d}\sigma^2 = \alpha^2(\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\,)$$

Here again space is non contractible. On the throat we get a minimum area $4\pi\alpha^2$. Here again there are no discontinuities in the geodesic system.

VI – ORBIFOLD STRUCTURE

Let's consider a 2D surface defined by its Riemanian metric with signature $(++)$

(14) $$ds^2 = \frac{dr^2}{1-\frac{\alpha}{r}} + r^2\,d\varphi^2 \quad with \quad \alpha > 0$$

Defined on $\{\,r\,,\varphi\,\} \subset \mathbb{R}^2$. One may consider, to keep ds positive, that this surface is defined for $r > \alpha$, that it is a bounded 2D manifold, bordered by a circle of perimeter $2\pi\alpha$. We can give an image of it in the form of a plane with a hole :

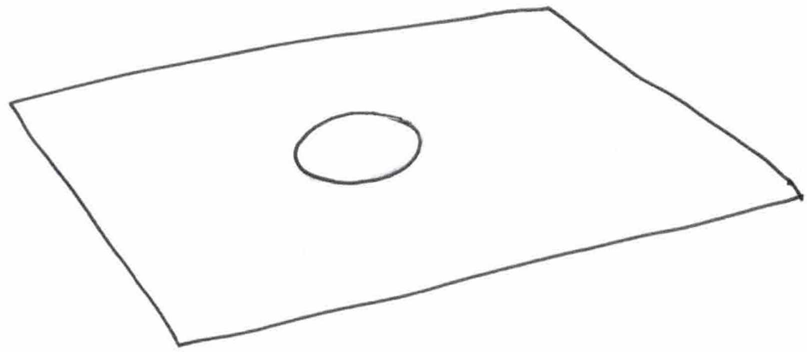


Fig.8 : A plane with a hole.

By implementing, for example, variable change (12) this metric becomes :

(15) $$ds^2 = \frac{1 + \operatorname{Log}\operatorname{ch}\rho}{\operatorname{Log}\operatorname{ch}\rho}\ \operatorname{th}^2\rho\, d\rho^2 - (\alpha + \operatorname{Log}\operatorname{ch}\rho)^2\, d\varphi^2$$

which evokes a bridge joining two planes through a throat circle with permiter $2\pi\alpha$:

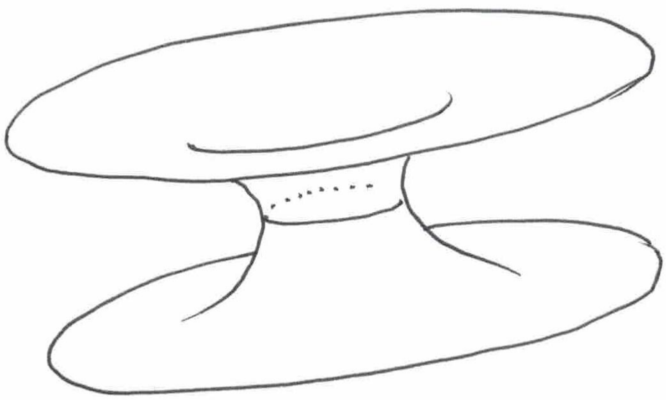


Fig.9 : Diabolo 2D

We can then identify the two layers and obtain a 2D orbifold structure:

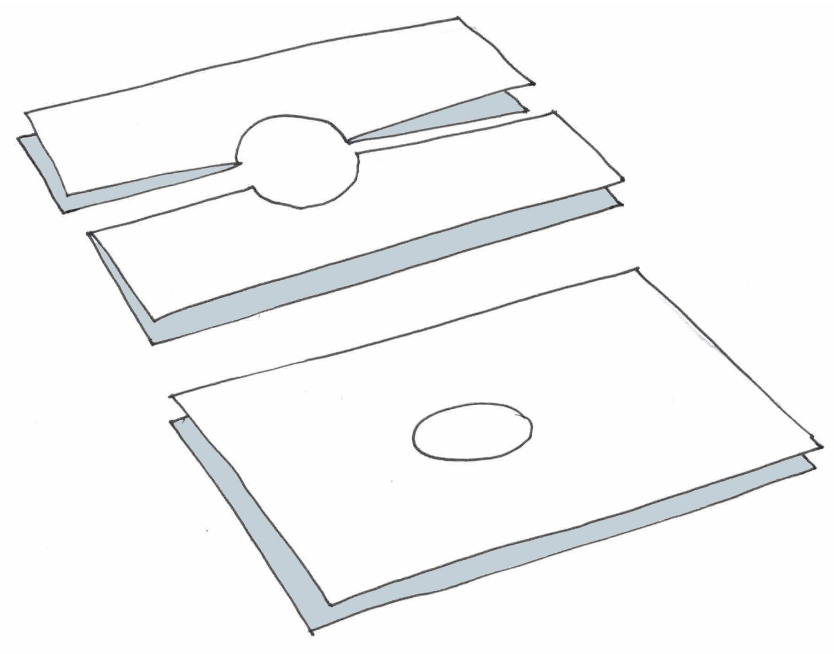


Fig.10 : Orbifold 2D

Note that this is simply introducing the concept of upside and downside on a 3D surface. This approach can be taken up again with the metric :

(16) $$ds^2 = \frac{dr^2}{1 - \frac{\alpha}{r}} + r^2\,(\,d\theta^2 + \sin^2\theta d\varphi^2\,) \quad with \quad \alpha > 0$$

with a similar result, i.e. a space bridge connecting two Euclidean spaces through a closed surface . Here again we will obtain a 3D orbifold structure where the throat circle is replaced by a throat sphere. Here again we can identify the points corresponding to $\pm u$ or $\pm\rho$, according to the change of variable considered and obtain an orbifold structure.

In 2D the orientation is defined by a direction of travel of the points of a triangle. When we drag this one (see figure 11) from one face to the other we obtain a grey triangle, enantiomorphic with respect to the first :

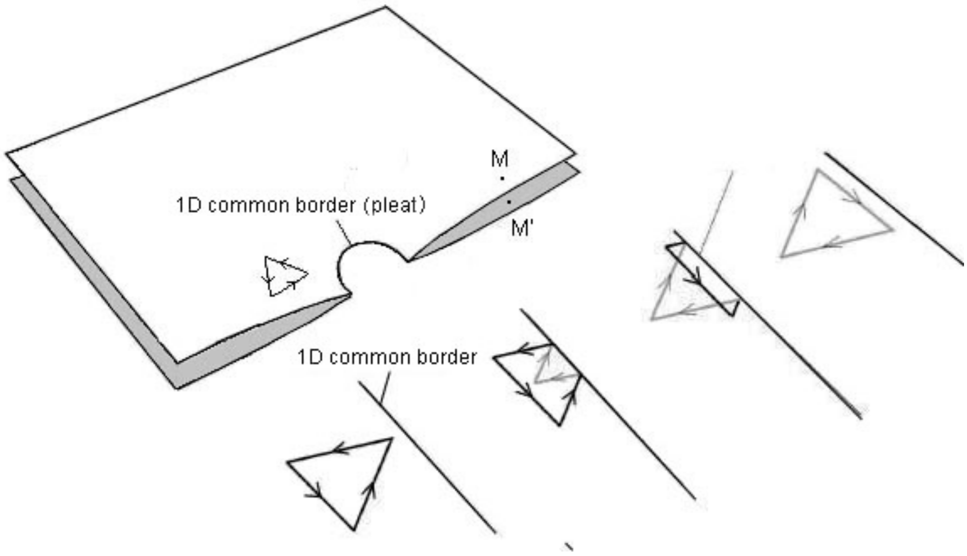


Fig.11 : 2D enantiomorphism.

The concept of front side and bach side speaks to intuition about a 2D object because it may be embedded a 3D Euclidean space. But we get puzzled with 3D objects. Anyway the space orientation corresponds to the direction of circling on the sides of a tetrahedron.

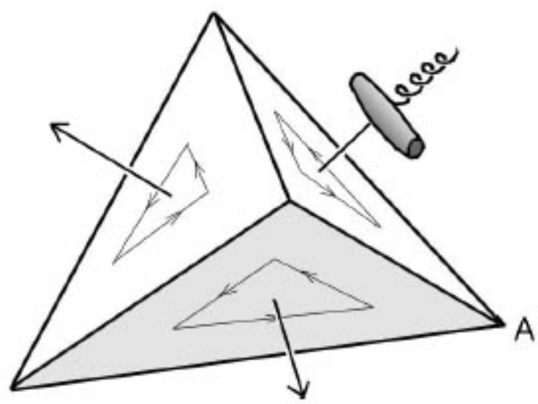


Fig.12 : 3D space orientation.

How to imagine the crossing of the 2D throat?

If we go back to the previous image the passage of the three points of the triangle is equivalent to the rebound of the summits-points on the portion of the throat circle considered. We will have the same phenomenon for the tetrahedron. We will compare its four summits to balls projected on a hard sphere. Consider the following figure.

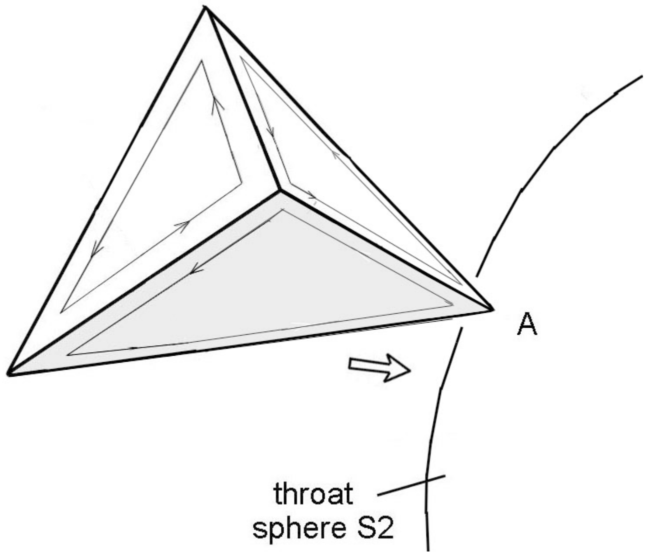
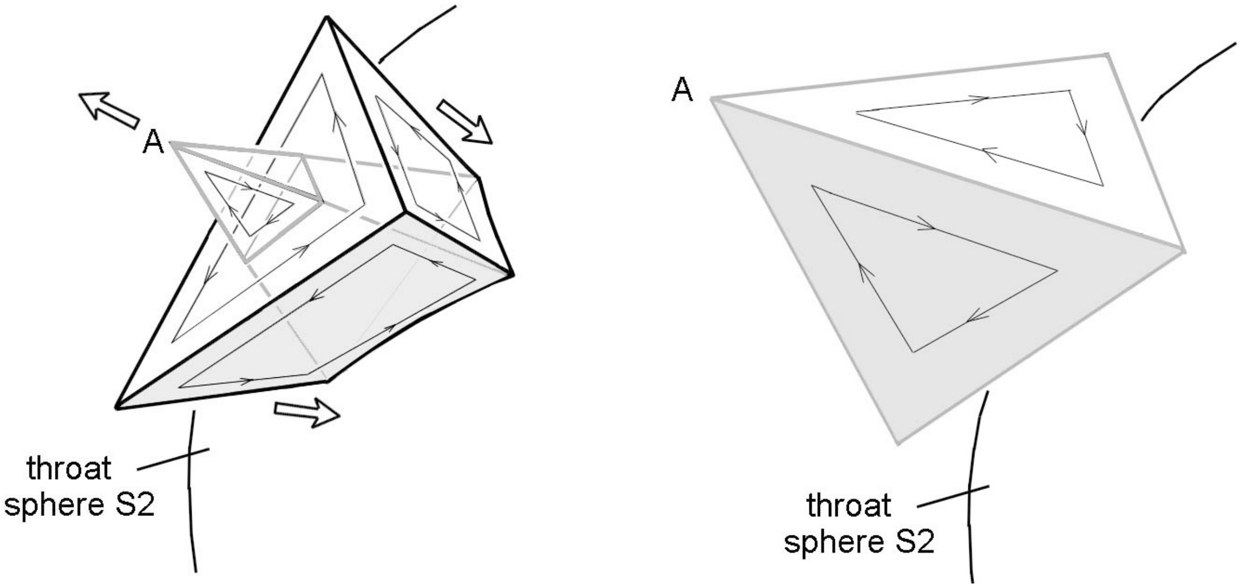


Fig.13 : The tetrahedron « falls » on the throat sphere.

This "ball" will be the first to "bounce" off the throat sphere:



Then the others will follow. In the end the tetrahedron will be "turned over", its orientation will be reversed. This tetrahedron "inhabiting the other side of this 3D hypersurface" will be represented in grey. It will be, with respect to the initial tetrahedron, in mirror, in enentiomorphics relationshi relation of enantiomorpohie with respect to this one. As a conclusion these 3D spaces are P-symmetrical.

!! We know that a space defined by a Riemannian metric is locally orientable if the determinant at this point is not zero. As the passage of the throat sphere achieves space inversion, it must be locally unorientable on the throat sphere. Therefore the determinant of the metric must be singular on this surface. With the change of variable (11) the metric (14) of this 3D hypersurface becomes

(17) $$ds^2 = \frac{1 + Log\, ch\,\rho}{Log\, ch\,\rho} h^2 \rho\, d\rho^2 + (\,\alpha + Log\, ch\,\rho\,)^2 (\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

The singularity manifests itself by the fact that the term tends on the throat sphere becomes the indeterminate form $\frac{0}{0}$.

!! Let us now return to the representation of the (unique) solution of Einstein's spherically symmetric equation as given by Schwarzschild in 1916, i.e. (5). This metric refers to a 4D hypersurface. In order to locally define (Gaussian) coordinates, i.e. to orient locally this hypersurface with respect to space and time, its metric must be locally non singular. If we consider that this solution refers to a space bridge (or wormhole) connecting two flat spaces through a closed surface we can see that the metric is doubly singular on the throat. Indeed on the throat surface $g_{tt} = 0$ and $g_{rr} = \frac{0}{0}$. Same conclusion when we choose to use the representation by means of the variables $\{\, t\,, \rho\,, \theta\, \varphi\,\}$. The two flat spaces connected by this groove surface are therefore PT-symmetrical. The passage of the test particles through this surface thus results in an inversion of both space and time.

VII – TOWARDS A PHYSICAL INTERPRETATION.

It is premature to attribute any physical significance to this solution of Einstein's spherically symmetric equation. Indeed, if we consider such an "object" as the result of the implosion of a mass under the force of gravity, it is impossible to imagine that such object would be free of rotation. So that the analyzis should start at least from Kerr's metric. .

However, we can give a physical meaning to this inversion of the time coordinate. As shown in 1970 by the mathematician Jean-Marie Souriau the inversion of time goes with with the inversion of mass and energy [19]. Thus these particles falling towards this gorge would have their mass and energy reversed.

Gravitationally, Schwarzschild's solution is synonymous with the force of attraction. If the masses are reversed when crossing the throat then the description of this wormhole should be considered to correspond, on the other fold to a repulsion and to coupled metrics :

(18-a) $$ds^2 = \left(1 - \frac{\alpha}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{\alpha}{r}} - r^2 (\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

(18-b) $$ds^2 = \left(1 + \frac{\alpha}{r} \right) dt^2 - \frac{dr^2}{1 + \frac{\alpha}{r}} - r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2)$$

In a system $\{ t , \rho , \theta , \varphi \}$ with the coordinate change $r = \alpha \left(1 + Log\, ch\, \rho \right)$

(19-a)

$$ds^2 = \frac{\mathrm{Log\,ch}\,\rho}{1 + \mathrm{Log\,ch}\,\rho} c^2 dt^2 - \frac{1 + \mathrm{Log\,ch}\,\rho}{\mathrm{Log\,ch}\rho} \alpha^2 \, \mathrm{th}^2 \rho \, d\rho^2 - \alpha^2 (1 + \mathrm{Log\,ch}\,\rho)^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

(19-b)

$$ds^2 = \frac{1 + \mathrm{Log\,ch}\,\rho}{2 + \mathrm{Log\,ch}\,\rho} c^2 dt^2 - \frac{2 + \mathrm{Log\,ch}\,\rho}{1 + \mathrm{Log\,ch}\,\rho} \alpha^2 \, \mathrm{th}^2 \rho \, d\rho^2 - \alpha^2 (1 + \mathrm{Log\,ch}\,\rho)^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Note that system (19-a) + (19-b) is definitively not a new solution. This set is identical to the pair (18-a)+(18-b), expressed using another coordinate system. The geodesics are identical and fit together perfectly.

VIII – CONCLUSION

Taking Karl Schwarzschild's article of 1916 to the letter, we show that what is considered in the literature as "Schwarzschild's solution" is only its linearized form, when introduced by him to stick with the approximate solution published by Einstein a few months earlier, to describe the advance of Mercury's perihelion. Thus the classical interpretation is only an extension of the solution in a complex space-time, which goes with the non-preservation of the signature.

The coordinates proposed by Einstein and Rosen in 1935 change from a bounded manifold structure to a wormhole-type geometry.

It is proposed to express Schwarzschild's solution, unique according to Birkhoff's theorem, in another set of coordinates that leads to the same topology, but provides a connection with a flat metric at infinity.

It is shown that the singular aspects of the metric on the throat sphere go with a double inversion of space and time when it is crossed. Thus the two fold are PT-symmetrical. Exploiting the theory of dynamic groups, according to which the inversion of time goes hand in hand with that of mass and energy, it is suggested that this geometry achieves a mass inversion through the throat.

Two steps remain to be taken in the attempt to fit physical reality.

- Leaving this spherical geometry by introducing the rotation

- Answer the question "Is the time variable chosen the appropriate choice? »

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28 février retour de la seconde version :	
Dear Dr. Michea,	Cher Dr. Michea,
Your submission to Modern Physics Letters A entitled "The time independent spherically symmetric solution of the Einstein equation revisited." requires extensive revision before it may be considered for peer-review.	Votre soumission à Modern Physics Letters A de l'article intitulé « The Time independent spherically symmetric solution of the Einstein equation revisited » demande une grande révision avant de pouvoir être pris en compte par un referee.
In particular,	En particulier
1. The current submission must summarize very briefly on the background and historical materials. Established findings could be stated with proper citations instead of presenting portions of the original papers as figures. The authors are advised to take reference from a research-type publication of a professional and indexed journal (e.g. Physical Review Letters of APS).	1. L'article soumis doit résumer brièvement le contexte et les éléments historiques. On doit faire état des éléments acquis avec les citations en question, et non en faisant figurer des extraits des papiers originaux et des figures. Les auteurs doivent suivre les modèles types correspondant aux publicatios des travaux de recherche (exemple Physical Review Letters de l'American Physical Society) On avait fait fait cela pour apporter des pruves concrètes de ce que nous évoquions.
2. The paper must distinguish clearly on the authors' original results, directly presenting their own work.	2. L'article doit permettre de dégager clairement les résultats originaux des auteurs en présentant d'emblée leurs résutats.
We regret that our journal office could not consider your paper for peer-review unless the necessary revisions are made.	Nous regrettons de vous dire que nous ne pouvons soumettre votre arricle à un referee si ces nécessaires révisions ne sont pas opérées.

Annexe D : Troisième rédaction de l’article

The time independent spherically symmetric solution of the Einstein equation revisited

Jean-Pierre Petit[6], Gilles d'Agostini[7], Nathalie Debergh[8] and Sebastien Michea[9]

Manaty Research group

Abstract :

From introducing a new coordinated change we first interpret Schwarzschild's solution as a wormhole connecting two Minkowskian spaces. Then we identify the symmetrical points of the two layers. Thus the throat sphere becomes a fold. In this new orbifold structure we show that the crossing of the fold is equivalent to PT-symmetry. After dynamic systems theory, T-symetry goes with energy and mass inversions so that such structure would achieve mass-inversion process. On geometric grounds mass inverted particles particles would be invisible to us.

Keywords : Non contractible hypersurface, throat sphere, wormhole, orbifold, negative mass, mass inversion, spherically symmetric solution

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I-INTRODUCTION

In 1915 the advance of Mercury's perihelia is the astronomical phenomenon that the theory must account for. Einstein publishes his equation and produces a linearized solution, able to account for this very slight modification of the Newtonian dynamics. In 1916 the mathematician Karl Schwarschild produces an exact non-linear solution [1] of this equation in the form of a metric expressed in coordinates $\{ t, x, y, z \} \subset \mathbb{R}4$. He also chooses, like Einstein, to express the signature of this Rianianian metric as $(+---)$.

Paradoxically, he does not express the solution with these coordinates, transformed into the set $\{ t, r, \theta, \varphi \}$ with

(1) $$r = \sqrt{x^2 + y^2 + z^2} \geq 0$$

but chooses to express it, using an auxiliary variable R, defined by :

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:

(2) $R = (r^3 + \alpha^3)^{1/3}$ which automatically implies $R \geq \alpha$

This gives the metric the well known form, expressed in a system $\{ t, R \geq \alpha, \theta, \varphi \}$:

(3) $$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \frac{dR^2}{1 - \frac{\alpha}{R}} - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad R \geq \alpha$$

In effect, as :

(4) $$r^3 = R^3 - \alpha^3$$

if we consider regions of space-time where $R < \alpha$, as Antoci [2] and Abrams [3] note, this would contradict (1).

In fact, as noted by C.corda [4] keeping the coordinates $\{ t, x, y, z \} \subset \mathbb{R}4$ as Schwarzschild had defined them we obtain :
(5)

$$ds^2 = \left[1 - \alpha(r^3 + \alpha^3)^{-1/3}\right]dt^2 - \frac{r^2 dr^2}{(r^3 + \alpha^3)^{2/3}\left[1 - \alpha(r^3 + \alpha^3)^{-1/3}\right]} - (r^3 + \alpha^3)^{2/3}(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

It is then easy to show that the form (3) is in fact only the linearization of (5) for $r >> \alpha$. Why dis Schwarzschild opt immediately for a linearized solution? Because at the time no one imagined that there could exist objects with a so huge concentration of matter such as the translation of the advance of perihelia recquires a non-linear solution.

In 1923 G. Birkhoff shows [5] that this solution of Einstein's equation, in vacuum and with spherical symmetry is unique. This solution can only take on different aspects depending on the unlimited choice of coordinate sets that one opts for.

We notice however that the hypersurface described by this metric is not contractile which corresponds to the foliation by sub-metric :

(6) $$d\sigma^2 = (r^3 + \alpha^3)^{2/3}(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

These spheres then have a minimal area $4\pi\alpha^2$. This leads Einstein and Rosen in 1935 [6] to propose the expression of metrics in coordinates by implementing a new coordinate system , $\{ t, u, \theta, \varphi \}$ throught the change :

(7) $$r = \alpha + u^2 = 2m + u^2$$

which gives the metric the form:

(8) $$ds^2 = \left[\frac{u^2}{u^2 + 2m}\right] dt^2 - 4\,(u^2 + 2m)\,du^2 - (u^2 + 2m)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

This gave birth to the concept of wormhole by imagining two layers, one corresponding to the positive values of u and the other to the negative values. As in the other expressions of the metric, the determinant is null on the Schwarzschild sphere, here considered as a throat sphere. It should be noted that according to this formulation the metric is not Minowskian, flat, at infinity. This object is given the name of wormhole and is the subject of many works ([7], [10]) .

II. SOLVING THE PROBLEM OF FLATNESS AT INFINITY

This problem of flatness at infinity can be solved by the following coordinate change :

(9) $$R = \alpha + \mathrm{Log\,ch}\,\rho$$

which gives :

(10)

$$ds^2 = \frac{\mathrm{Log\,ch}\,\rho}{1 + \mathrm{Log\,ch}\,\rho} c^2 dt^2 - \frac{1 + \mathrm{Log\,ch}\,\rho}{\mathrm{Log\,ch}\rho}\ \mathrm{th}^2\rho\, d\rho^2 - (\alpha + \mathrm{Log\,ch}\,\rho)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

When $\rho \to \pm\infty$ the metric tends to :

(11) $$ds^2 = c^2 dt^2 - d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

which is what it's all about.

On the throat $g_{tt} = 0$ and the term $g_{\rho\rho}$ gets an indeterminate form $\frac{0}{0}$. In the vicinity to the throat an expansion into a series of the functions $ch\,\rho$ and $sh\rho$ shows $g_{\rho\rho}$ that tends to 2. Here again we get a wormhole structure. The degenerated metric is :

(12) $$d\sigma^2 = \alpha^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Here again space is non contractible. On the throat we get a minimum area $4\pi\alpha^2$. There are no discontinuities in the geodesic system. But on the throat sphere, in this representation we observe a double degeneration through the terms g_{tt} and $g_{\rho\rho}$. In what follows we will try to give a physical meaning to this property.

III. **FROM WORMHOLE TO AN ORBIFOLD GEOMETRY**

Classically we consider that a wormhole represents a communication either between two spaces F1 and F2 or putting in communication two distant regions of the same space F. See figure 1.

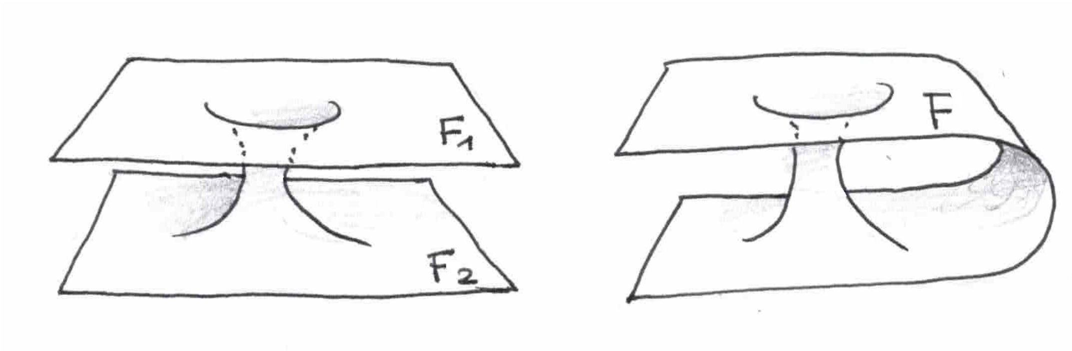

Fig.1 Schematic representation of a Wormhole

By keeping only the spatial part of representation (10) of the metric we get:

(13)

$$d\sigma_a^2 = \frac{1 + \mathrm{Log\,ch}\,\rho}{\mathrm{Log\,ch}\rho}\ \mathrm{th}^2\rho\, d\rho^2 + (\alpha + \mathrm{Log\,ch}\,\rho)^2 (\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

By limiting itself to two dimensions :

(14)

$$d\sigma_b^2 = \frac{1 + \mathrm{Log\,ch}\,\rho}{\mathrm{Log\,ch}\rho}\ \mathrm{th}^2\rho\, d\rho^2 + (\alpha + \mathrm{Log\,ch}\,\rho)^2\, d\varphi^2$$

To make things easier, remember that the 2D metric (14) is obtained by applying the variable change (9) to the metric (15):

(15)
$$d\sigma_2^2 = \frac{dR^2}{1 - \frac{\alpha}{R}} + R^2 d\varphi^2$$

In this representation $\left\{ R \geq \alpha\,, \varphi \right\}$ it describes a surface which can then easily be represented as imbedded in R3 $\left\{ R \geq \alpha\,, \varphi\,, Z \right\}$ and then corresponds to a parabola rotating around an axis of symmetry. We then have two planes connecting through a throat circle. See figure 2.

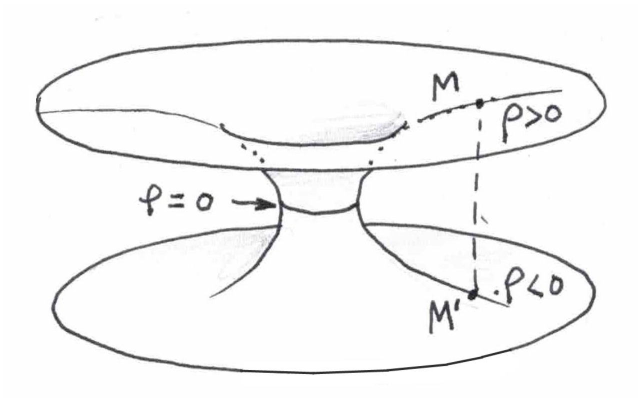

Fig .2 : Imbedded 2D diabolo in 3D space

The parabola equation being:

(16) $$Z = \pm 2\alpha\sqrt{\frac{R}{\alpha} - 1}$$

Although a representation by imbedding is no longer possible, we can consider the 3D equivalent metric (13) knowing that this one derives from the expression (17) through the coordinate change (9).

(17) $$d\sigma_2^{\,2} = \frac{dR^2}{1 - \frac{\alpha}{R}} + R^2(\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

We will be brought into coincidence corresponding points $\rho > 0$ and $\rho < 0$. In 2D it is again possible to imbed the object in R3 , as showns of figure 3. The result is a fold along a closed surface.

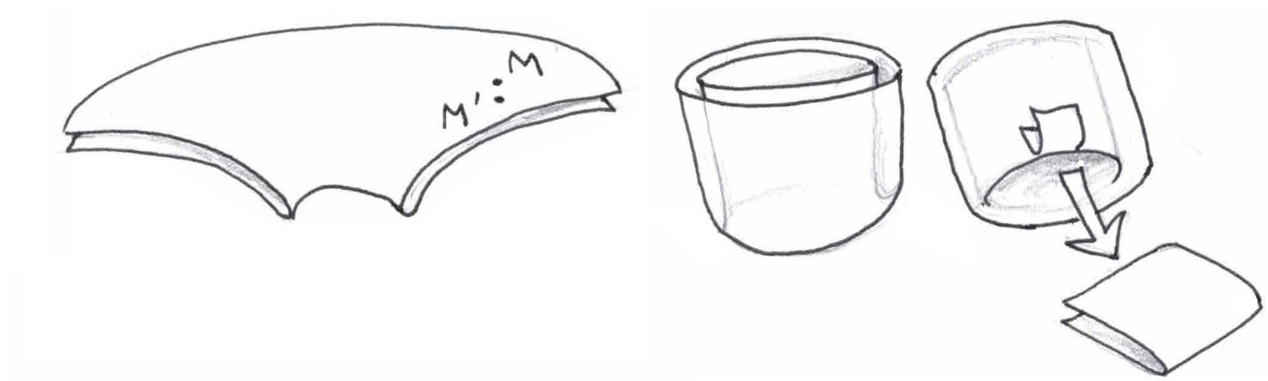

Fig.3 : Identification of symmetrical points

On the right is a neighborhood of the circle along which this fold is operated.

We are now going to orient this surface by drawing triangles A B C with an arbitrary orientation. If we consider that these triangles are like decals we could drag this triangle. We then see that this puts in coincidence two enantiomorphic triangles, of opposite orientations, that is to say that this operation is equivalent to a P-symmetry. See figure 4 where the triangle which has "passed to the other side" has been shown in grey.

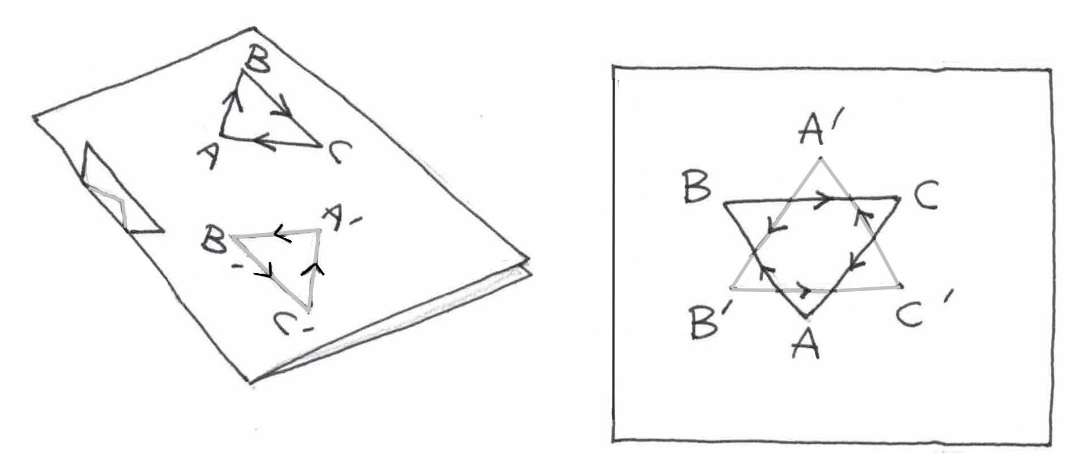

Fig.4 : 2D Inversion

It can be seen that the operation, which does not introduce any discontinuities in the geodesics, reverses the orientation of the triangle. One could also represent this triangle as three balls that would converge towards the "circle-fold" by "bouncing" on it. As ball A would bounce first, the triangle formed by the three balls would still be inverted.

We will consider performing the operation in 3D by giving an (arbitrary) orientation to the four faces of a tetrahedron. See figure 5.

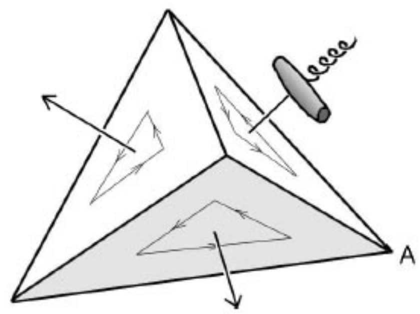

Fig. 5 : Oriented tetrahedron

It is then convenient to represent this tetrahedron as a set of four balls projected onto a hard sphere and bouncing off it. This image of the rebound illustrates a fold in 3D space, this time along a sphere.

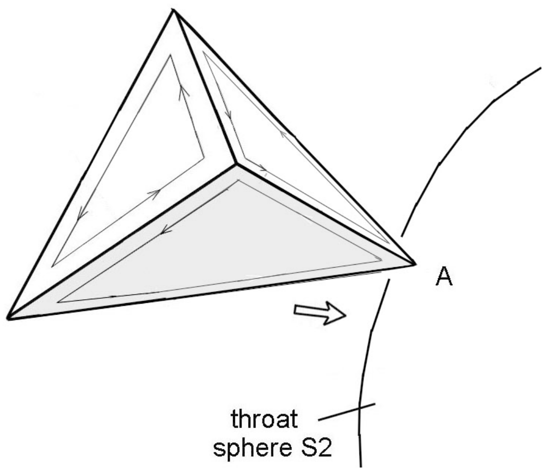

Fig.6 : Fig.6: The four "balls" fall on the sphere which represents a 3D fold.

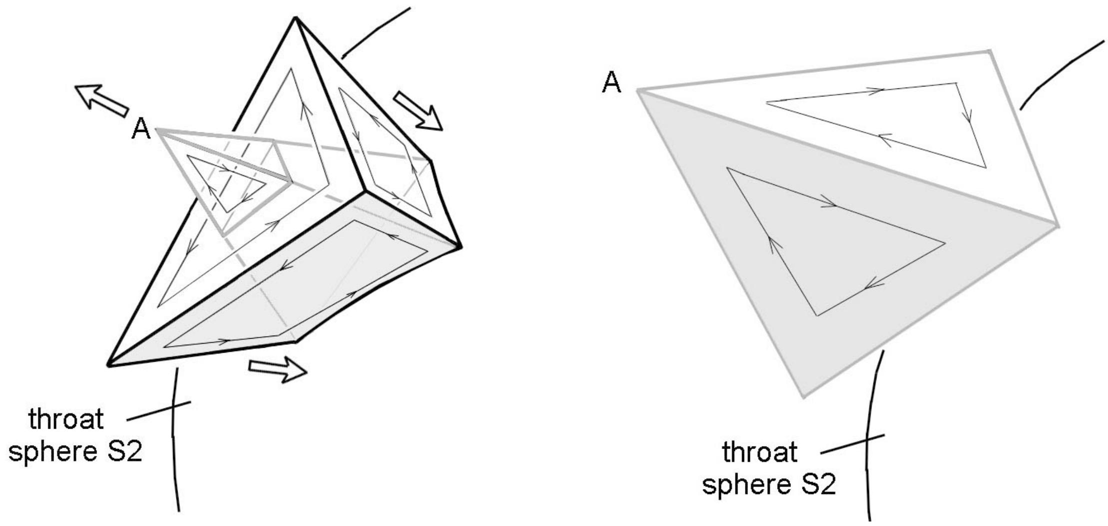

Fig.7 : The crossing of the 3D fold indicates a P-symmetry.

This geometrical structure is no longer a manifold but an orbifold, i.e. a geometrical shape including a singular region (where it is no longer possible to define coordinates, here along a closed surface). This can also be read in the metric (13) where we can see that the term $g_{\rho\rho}$ becomes $\frac{0}{0}$ in this region $\rho = 0$.

Returning to a 2D object, we can graft on it a 3D space by considering the normal to the surface as a third coordinate a t-coordinate. When the fold is crossed, the normal tilts and adopts an opposite direction. If we consider that this geometrical structure achieves a coincidence of two 3D (Euclidean) spacetimes through the crossing of this fold, in this orbifold structure translates a double inversion of space and time, a PT-symmetry.

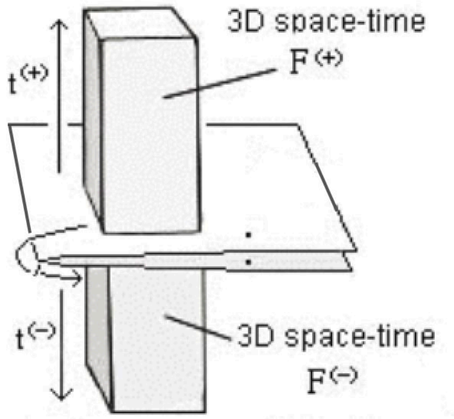

Fig.8 : PT-symmetry by crossing a fold

Returning now to the equivalent interpretation of the geometry associated with metrics in its the classical form $\left\{ t , R \geq \alpha , \theta , \varphi \right\}$:

(18) $$ds^2 = \left(1 - \frac{\alpha}{R}\right) c^2 dt^2 - \frac{dR^2}{1 - \frac{\alpha}{R}} - R^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

we can envisage that this structure is no longer a wormhole but a region of space where a double inversion of space and time takes place (PT-symmetry), by crossing what was previously considered as a throat surface but which then becomes the singular region of an orbifold (where the determinant of the metric is zero).

The question of the continuity of geodesics has been examined in reference [16].

The geodesics of the Schwarzschild's metrics with α > 0 figure trajectories of test-particles masses undergoing the attraction of a mass

(19) $$M = \frac{\alpha c^2}{2G} > 0$$

This object is thus "attractive". For the sake of conformity with a physical vision of things, it would be logical that the geodesics, expressed in the system of coordinates $\left\{ t , R \geq \alpha , \theta , \varphi \right\}$ would no longer being interpreted as inscribed "in two sheets" but woud correpond to a "bounce" in a single sheet, though a combination of an attraction and repulsion after bouncing. This corresponds to the metric :

(20) $$ds^2 = \left(1 + \frac{\alpha}{R}\right) c^2 dt^2 - \frac{dR^2}{1 + \frac{\alpha}{R}} - R^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

Thus "the object" does not have the same appearance, with respect to the test-masses, according to whether they follow one or the other of the two families of geodesics. This leads to a bimetric conception of space-time ([17], [18], [9]).

The continuity of geodesics is not a problem [16] . In the system this is equivalent to associating the metric (as transformed from (18) into (19) through (9)):

$$(21)\quad ds^2 = \frac{2+\mathrm{Log\,ch}\,\rho}{1+\mathrm{Log\,ch}\,\rho} c^2 dt^2 - \frac{1+\mathrm{Log\,ch}\,\rho}{2+\mathrm{Log\,ch}\,\rho}\ \mathrm{th}^2\rho\, d\rho^2 - (\alpha + \mathrm{Log\,ch}\,\rho)^2 (\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

We can then wonder about the physical meaning of this inversion of space and time.

In quantum field theory [20] we have space and time inversion operators. Classically one chooses to opt for an antisymmetric and antilinear T-operator, to avoid taking into account negative energy (and mass) states, considered a priori impossible. But as shown by N.debergh [21] quantum formalism does not exclude the existence of such objects in our space-time.

In the theory of Dynamical Systems [22] the mathematician Jean-Marie Souriau also shows, on the basis of the study of the coadjoint action of the complete Poincaré group Minkowski spacetime, that the inversion of the time coordinate leads to the inversion of energy and mass.

IV- CONCLUSION

It can therefore be considered that this interpretation of Schwarzschild's solution would reflect a region of space where the inversion of mass and energy of the particles would take place. This would escape observation for two reasons. If we suppose that negative mass particles emit photons of negative energy, then we can say that our measuring instruments (and our eyes) are not reacting to these signals.

But we can also say that negative energy photons travel on negative geodesics, which do not interfere with those of the positive world. The invisibility of such objects would then emanate from simple geometrical considerations.

The interaction between masses of opposing signs could then only be achieved by antigravitation in accordance with our biometric Janus Cosmological Model ([17], [19]).

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[21] N.Debergh, J.P.Petit and G.D'Agostini : Evidence of negative energies and masses in the Dirac equation through a unitary time-reversal operator. , J. Phys. Comm. **2** (2018) 115012

[22] J.M.Souriau: Structure des systèmes dynamiques. Dunod Ed. France, 1970 and Structure of Dynamical Systems. Boston, Birkhaüser Ed. 1997. For time inversion see page 190 equation (14.67).

Réaction de la revue à cette troisième version :

We regret that your submission titled "The time independent spherically symmetric solution of the Einstein equation revisited." deviates from our journal's publishing policy and academic guidelines. Your submission is hereby returned to you and we hope you find better results with more suitable journals. We thank you for your interest in our journal.	Nous avons le regret de vous informer que l'article que vous avez soumis à notre revue intitulé « Réinterprétation de la solution stationnaire et à symétrie sphérique de l'équation d'Einstein ». ne correspond pas à notre politique éditoriale et à notre ligne de conduite académique. Nous rejetons donc votre artucle en vous souhaitant un meilleur résultat et soumettant votre article à des journaux plus adéquats. Nous vous remercions de l'inérêt que vous portez à notre journal.

13 mars 2020

En attente de notre dernier message, date du 4 mars 2020 :

- I object. It is clear that the person who made this summary and absurd decision simply did not read this third version of our article and cannot be the referee who, after reading the second version of our article, had asked for its strong points to be made clear, in a more concise manner, which we did. We ask for a serious examination of this third version by a competent referee. If this decision of rejection were to be upheld, then your newspaper would never have published Einstein and Rosen's 1935 article, built, like ours, on a simple change of variable applied to the solution published by Karl Schwarzschild in 1916.	- Nous protestons. Il est clair que la personne qui a pris cette décision sommaire et absurde n'a simplement pas lu cette troisième version de notre article et ne peut pas être le referee qui, après avoir lu la deuxième version de notre article, avait demandé que ses points forts soient mis en évidence, de manière plus concise, ce que nous avons fait. Nous demandons un examen sérieux de cette troisième version par un referee compétent. Si cette décision de rejet devait être maintenue, votre journal n'aurait jamais publié l'article d'Einstein et Rosen de 1935, construit, comme le nôtre, sur un simple changement de variable appliqué à la solution publiée par Karl Schwarzschild en 1916.